

## Do small cities need more public transport subsidies than big cities?

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### *Abstract*

This paper compares the optimal public transport subsidies for a representative bus corridor in a small city (Karlstad) and in a big city (Stockholm) in Sweden. In the smaller city, the optimization of the fare is more important for welfare, whereas in the larger city, the frequency is more important. Moreover, the optimal subsidy is higher in small than in large cities because of larger positive externalities of reduced waiting and schedule delay costs, whereas the negative crowding externalities are larger in the large city. The bus subsidy is important for the income distribution in the small city.

*Keywords:* Cost-Benefit Analysis, Transport plans, Public transport, Subsidies, External costs, Fares, Frequencies, Bus stops, Redistribution

*JEL Codes:* R41, R42, R48

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## **Do small cities need more public transport subsidies than big cities?**

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### ***Abstract***

This paper compares the optimal public transport subsidies for a representative bus corridor in a small city (Karlstad) and in a big city (Stockholm) in Sweden. The optimal subsidy is derived as a result of first best pricing, frequency, bus stop spacing and bus lane policies. The optimal cost-recovery of the buses depends on the relative size of two costs: waiting time (the Mohring effect) and on crowding/congestion. In the big city the high crowding cost is dominating, approaching full cost-recovery in the first best optimum. In the small city, however, the Mohring effect dominates, implying larger subsidies in optimum. These results continue to hold under several policy constraints: absence of road tolling, no frequency optimization, no bus lanes and mild marginal cost of public funds. The bus subsidy is also more effective as a redistribution policy in the small city.

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*Keywords:* Cost-Benefit Analysis, Transport plans, Public transport, Subsidies, External costs, Fares, Frequencies, Bus stops, Equity

# 1 INTRODUCTION

Efficient public transport systems are key for the accessibility in metropolitan areas. This paper adds to the literature by comparing the optimal subsidies (and fares), frequencies and bus stop spacing, for two real bus corridors: the busiest inner-city bus line in Stockholm (population of 1.7 million) and a representative bus corridor in the small city of Karlstad in Värmland (population 60 000 inhabitants). The optimal subsidies are computed using the same formal model for two cities. The model is calibrated using the reactions to the Stockholm toll implementation. The model is used for a wide range of policy assumptions: with or without tolling of cars, with or without frequency optimisation, with higher marginal cost of public funds and with different cross-price elasticities. This allows to compare our study with results for other cities.

Optimal pricing, frequency, and spacing of bus stops have been extensively studied, analytically, empirically and by model simulations (Parry and Small, 2009; Basso and Silva2014; Kilani et al., 2014; Tirachini et al., 2014). However, the focus has been on large and congested cities. Optimal subsidies, frequency and bus stop spacing for smaller cities are

much less studied.<sup>1</sup> The studies for smaller cities are mostly analytical rather than based on real case studies. Jansson et al. (2015) study the relationship between optimal bus fares and demand elasticity analytically and with model simulations, and Tirachini et al. (2010) also consider a range of demand densities in the transit network (including low density) focusing on the costs of different types of transit with high fixed costs such as heavy rail, light rail and bus rapid transit (BRT).

The two corridors under study are broadly representative of large and small cities in Europe, with the exception that Stockholm has congestion charges, and this is important as the introduction of congestion charges allows to study the effectiveness of road tolls and by symmetry, the effectiveness of public transport prices to address road congestion. The Stockholm corridor has a large market share of bus, car and cycling of 54%, 15%, and 31%, respectively. For Karlstad the corresponding market shares are 17%, 61% and, 22% making it a car city. Our main contribution is that we use the exact same transport model in the big and the small city and make the input assumptions consistent between them which enable a fair comparison. In both cities, cycling accounts for a substantial market share, but the design of the bus stop is different. In Stockholm, the bicycle lane often disappears at the bus stop, forcing cyclists to divert into the car lane. But in Karlstad the bicycle lane continues around the bus stop. Exploring the external effect of cycling in the two cities is a further contribution to the literature. In this paper we extend the Börjesson et al. (2018) model to include also the whole off-peak period, since this allows to make a more thorough

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<sup>1</sup> In rural areas with low density, paratransit or demand responsive transport (DRT) is often organized (instead of having fixed bus lines) to provide mobility for inhabitants. However, DRT is often difficult and costly to operate and many of them failed (Enoch et al, 2006). Moreover, the optimization of DRT is very different from fixed bus lines because it involves the endogenous pickup and drop off points (in Markovic et al. (2015), for instance). For these two reasons, the literature on rural areas are less relevant to this paper.

comparison of the optimal bus systems for the two cities and to have a more nuanced view on the need for subsidies.

High optimal transit subsidies for several metropolitan areas (London, Santiago, etc.) have been proposed by Parry and Small (2009), Basso and Silva (2014). The high subsidy result of Parry and Small (“P&S”) (2009) for buses is driven by the absence of road tolls and a high diversion ratio (50% to 85% in the peak) between public transport and car use, combined with scale economies for bus. In Basso and Silva (“B&S”) (2014), the high bus subsidies are also driven by the absence of road tolls and the good substitutability of bus and car. When bus and car are good substitutes, a high bus subsidy does as well as a road congestion toll as long as the MCPF is one. For a wider selection of cities, Cervero and Guerra (2011) estimate that 18 out of 25 systems in 24 US cities have subsidies exceeding half of the operating expenses. However, higher subsidies (as percentage of operating costs) can be found in areas of lower density, primarily in cities in developed countries (Cooke and Behrens, 2017).<sup>2</sup> In Sweden more than half of the public transport supply (and subsidies) can be found in small cities or rural areas. Getting subsidies right in small cities is therefore key to policy makers.

Optimal pricing and frequency principles for public transport were developed by Mohring (1972), Jansson (1984, 1980), Viton (1980), Kraus (1991), Jara-Díaz and Gschwender (2003), Parry and Small (2009), Monchambert and de Palma (2014), and De Borger and Proost (2015). There are several factors determining the optimal level of subsidies in public transport, and many of these have different impacts in small and large cities. First, for rail-based public transport the marginal cost is lower than average cost, because of high fixed costs (in particular for rail, tram and metro). A second reason for subsidizing public transport is that the

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<sup>2</sup> Studying the viability of transit systems, they analyse the subsidies of the transit operation costs in 17 cities in developing countries and 30 cities in developed countries. They find that cities in the developing countries have lower (higher) transit subsidies (in percentages) than the developed cities when demand density is low (high).

externalities from car use (emissions, congestion, noise and accidents) are not fully internalized and that bus and rail can be a good substitute. For many reasons the externalities from car use are larger in a bigger city, particularly during the peak. Third, pricing of public transport has positive economies of density: more users allows higher frequency, implying decreased waiting costs<sup>3</sup> (Mohring, 1972). Fourth, crowding in public transport from more passengers (de Palma et al., 2015), and road congestion from higher bus frequency (Tirachini et al., 2014) induce a negative external effect of higher bus use. These effects are relevant primarily in dense cities. We find that the optimal subsidies as a share of the total operation cost are substantially larger in the small city than in the bigger city, so no or little public bus subsidy can be an optimal result for a big city. This result is robust under many assumptions like the impossibility to charge congestion tolls for cars.

These results stand in contrast to the results of the model simulations by Jansson et al. (2015), showing that the fares and subsidy rate should be approximately the same in low- and high-density cities. The difference compared to our study is that Jansson et al. assume smaller (12 seats) buses in the small city and that the demand varies so much in time and space that crowding occasionally arises also there. We, however, assume that the standard bus size is optimal in the small city. We justify these assumptions by the observation that small buses are not so much cheaper than standard sized buses because the driver counts for a large share of the operation costs. Moreover, even if the demand varies over space there are in fact no severe problems with passengers not being able to board the bus due to crowding. For Stockholm we assume the largest possible buses, with ribbed sections, that can still turn at the radius required

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<sup>3</sup> While van Reeve (2008) claimed that without subsidization the profit-maximizing frequencies are at least as high as the welfare-maximizing frequencies, Basso and Jara-Díaz (2010) argued that a more realistic setting will result in the profit-maximizing frequencies being lower than the welfare-maximizing frequencies and there will be welfare losses without subsidization. Thus, the Mohring effect provides an argument for subsidization.

in the city. But also in Stockholm, it is unusual that passengers cannot board the bus due to crowding.

Our finding that large subsidies are less necessary in bigger cities also stands in contrast to the results of P&S (2009) and B&S (2014), finding that large subsidies can be justified. There are three reasons for our contrasting result. The first is that Stockholm already has congestion pricing and bus lanes. More important still is that the cross-elasticity between bus and car is very high in P&S and B&S, (such high elasticity is not applicable in a city where the share of bus trips is 60%, while the share of car is 20%). A final reason for the difference in results is that we take crowding in the buses and congestion in the bus lane into account while this factor is absent in B&S and less important in P&S.

In contrast to what is found for other big cities, we find however that the subsidies are more substantial in Karlstad, due to less crowding and congestion implying low marginal costs of more bus users implying low fares. It is the Mohring effect, trading-off the waiting and schedule delay costs against bus operation costs (Mohring, 1972) that ensures that the bus frequency is kept reasonably high at three to four buses per hour.

Another contribution of this paper is that we investigate the sensitivity of the assumptions of the marginal cost of public funds (MCPF) to cover the deficit. MCPF for Sweden is high, 1.3 (Sørensen, 2010), which is not surprising given that MCPF is higher in the Scandinavian countries than in most other countries (Kleven and Kreiner, 2006). However, we also consider that some trips are commuting trips and for these the distortion on the labour market is the major component of the MCPF (Parry and Bento, 2001), implying a lower MCPF. Since the MCPF depends on the share of commuters and the elasticity of labour supply with respect to generalized commuting costs, which is uncertain, the sensitivity with respect to MCPF is important. We find that the MCPF has a substantial impact on the optimal subsidies, in particular in Karlstad.

In his seminal work, Mohring (1972) explicitly models the optimal number of bus stops as a trade-off between the operators' costs and in-vehicle travel time versus access time. This is consistent with Tirachini (2014), who finds that the optimal number of bus stops per km increases with density. We also find that the optimal distance between bus stops is shorter in the denser city. The reason is mainly that in the small city there is no bus lane and a larger market share for cars, so the bus stops delay car traffic more frequently. Moreover, the average bus stopping time per boarding and alighting is higher with fewer passengers at each stop. The present spacing of bus stops is quite close to optimal, in the Stockholm corridor, but too short in the Karlstad corridor.

To the extent that buses are more intensively used by low income citizens, subsidies to buses could be justified as an income redistribution policy or a right to accessibility. Indeed, we find that the income redistribution effect is large in the small city, because of large differences in the mode choice (and to some extent trip frequencies) between high and low-income travelers. In Stockholm, however, the redistribution effects of bus subsidies are small because high and low-income travelers have similar mode choice and trip frequencies.

## 2 THE MODEL

Section 2.1 sets up a small theoretical model demonstrating how the optimal subsidy depends on different factors. In the next sections 2.2 to 2.7 we lay out the simulation model that is used to compute optimal fares and optimal subsidy rates for Stockholm and Karlstad.

### 2.1 The optimal subsidy rule

The cost recovery theorem for roads is well known: proposed by Mohring and Harwitz (1962) and reviewed by Small and Verhoef (2007). This theorem had been discussed for public transport with waiting time (de Palma and Lindsey, 2007) and crowding by Hörcher and Graham (2018), indicating that cost recovery decreases with increasing waiting time costs and increases with crowding externalities.

In a first best world, where car trips are correctly priced, we can derive a simple expression for optimal fare and frequency. When we consider only one uniform demand period within the day, we can focus on the net welfare of public transport trips, defined as:

$$\Omega = \int_0^q P(q) dq - q \cdot (cr(q, f) + w\left(\frac{1}{f}\right)) - k(f) - e(f), \quad (0)$$

where  $q$  is the demand density,  $P(q)$  is the inverse demand function and  $f$  represents the bus service frequency. The user cost ( $uc$ ) is a function of crowding costs  $cr(q, f)$  and schedule delay and waiting costs  $w\left(\frac{1}{f}\right)$  (we assume that the access and in-vehicle times are constant) and  $k(f)$  is the operating and lease cost of buses. The external cost of the bus frequency  $e(f)$  consists of the delay to other road vehicles, air pollution and accidents. In equilibrium the marginal benefit equals the user cost,  $P(q) = \tau + uc$ . Using this and optimizing  $\Omega$  with respect to the number of trips and frequency, respectively, gives

$$\tau = q \cdot \frac{\partial uc}{\partial q} = q \cdot \frac{\partial w}{\partial q} + q \cdot \frac{\partial cr}{\partial q} = q \cdot \frac{\partial cr}{\partial q} \quad (0)$$

$$-q \cdot \frac{\partial uc}{\partial f} - \frac{\partial e}{\partial f} = \frac{\partial k}{\partial f}. \quad (0)$$

The optimal fare equals the marginal external crowding cost, since  $\frac{\partial w}{\partial q} = 0$ . The optimal frequency trades off the reduction in crowding, waiting and schedule delay cost ( $q \frac{\partial uc}{\partial f}$ ) with the additional operation, capital and external cost ( $\frac{\partial k}{\partial f} + \frac{\partial e}{\partial f}$ ). Assuming a that the crowding cost,  $cr$ , is homogenous of degree 0 in  $q/f$  (such that Euler's Theorem gives  $q \frac{\partial cr}{\partial q} = -f \frac{\partial cr}{\partial f}$ ) and an operation and capital cost that is proportional to frequency, the net revenue of public transport in the first best is

$$\frac{R}{q} = \tau - \left(\frac{f}{q}\right) \frac{\partial k}{\partial f} = q \cdot \frac{\partial cr}{\partial q} + \frac{\partial k}{\partial f} \left(\frac{f}{q}\right) = -f \cdot \frac{\partial cr}{\partial f} + f \cdot \frac{\partial uc}{\partial f} + \left(\frac{f}{q}\right) \frac{\partial e}{\partial f} = f \frac{\partial w}{\partial f} + \left(\frac{f}{q}\right) \frac{\partial e}{\partial f}. \quad (0)$$

We see that the net revenue per passenger decreases with the waiting and schedule delay externality (“Mohring effect”) but increases with the other externalities (road congestion, air

pollution, accidents) associated to a higher frequency. The crowding externality no longer appears explicitly in the net revenue function because it has been used to determine the optimal frequency (Euler's theorem). To make the model more complete, we add a peak and an off-peak period that represent  $n_p$  and  $n_o$  hours per year, respectively, and add the possibility that road congestion charges cannot be introduced for cars. This gives the net revenue function that will be the basis of our results:

$$\frac{R}{Q} = \sum_{i=p,o} \left[ f_i \frac{\partial w_i}{\partial f_i} - MEC_c^i \cdot d_{cb}^i + \frac{f_i}{q_i} \frac{\partial e_i}{\partial f_i} \right] s_i \quad (0)$$

where  $Q$  is the total number of standardized trips in a year and where  $s_i$  stands for the share of trips in peak and off-peak period. The term  $-MEC_c^i \cdot d_{cb}^i$  appears only in the absence of a car toll, and stands for the optimal second-best fare reduction to curb excessive road congestion. The net revenue per passenger will be a weighted average of peak and off-peak results. As above the crowding externality is absent from the net revenue function because it has been used to determine the optimal frequency.

Consider first the peak period, where frequency tends to be high to deal with crowding. A high frequency makes the waiting and schedule delay externality small ( $\frac{\partial w}{\partial f} \leq 0$ ) but the other externalities of frequency (road congestion) are higher in the peak. This tends to increase the net revenue. This reduction equals the marginal external congestion cost of a car times the diversion factor from car use to bus use when the bus fare is decreased. A high diversion factor (like the ones used in P&S (2009) and B&S (2014) tends to lead to high subsidies. And if bus and car are perfect substitutes and the total number of trips is inelastic, only the price difference between modes matters and one can achieve a balanced public transport budget by raising the congestion charge high enough (Basso & Jara-Diaz, 2012).

Consider now the off-peak period, when road congestion is less important and optimal frequency is also much smaller. Then the waiting and schedule delay externality becomes more important and leads to higher optimal subsidies.

We can now turn to the importance of the size of the city. In a larger city, densities tend to be higher, so frequencies will be higher as well as the external congestion costs and the air pollution costs. This means that, certainly in the peak period, the subsidy rate that is necessary will be lower. In a smaller city, frequencies will be lower, the waiting time externality will be more important and road congestion less important so that the subsidy rate needs to be higher.

Table 1 presents the results of the full model that we used to assess the first best subsidy rate in Karlstad and in Stockholm and these results are in line with the theoretical expectation.

Table 1: Degree of optimal self-financing for the bus corridors in Karlstad and Stockholm.

| City                                | Karlstad                                                      | Stockholm                                                     |
|-------------------------------------|---------------------------------------------------------------|---------------------------------------------------------------|
| Demand density                      | Low                                                           | High                                                          |
| Dominating components of user costs | Waiting and schedule delay                                    | Crowding, road congestion                                     |
| Degree of self-financing            | Deficit (83% of operating cost)                               | Small deficit (6% of operating cost)                          |
| Optimal peakbus fares               | 0.3 Euro/trip for short trips<br>0.6 Euro/trip for long trips | 0.6 Euro/trip for short trips<br>0.8 Euro/trip for long trips |

## 2.2 Supply and demand

We continue by formally defining an aggregate model for an urban area around a given bus corridor. We assume that the population is homogenously spread along the corridor. We use three different user groups, distinguished by income, indexed by  $k=1,2,3$  (high, mid, low). The travellers' preferences differ by user group but are assumed to be homogenous within the user group. Travelers in the three income groups therefore have different values of time. We assume that there is a separate bicycle lane in the corridor.

The users' marginal costs of travelling consist of the pecuniary and non-pecuniary costs. The time costs of a bus trip include the in-vehicle time cost, the waiting and schedule delay cost at

bus stops, and the access time cost to bus stops. Since the modes interact at bus stops, the frequency of buses and number of bus stops affect the travel time of cars and cyclists. The time cost of the bicycle trip depends on the congestion on the bicycle lane, and in case the bicycle lane disappears at the bus stop,<sup>4</sup> also on the interaction with the other modes.

The marginal cost of bus supply depends on frequency, the congestion on the bus lane, and the number of bus stops. One more bus stop increases supply cost because each bus stop has an investment and maintenance cost and it slows the bus down, which in turn increases costs of fuel and drivers. We also include external costs caused by health-damaging air pollution emissions in the welfare function. The carbon dioxide emissions are left out of the model because they are internalised in the fuel tax.

We limit our analysis to one corridor rather than modelling a full network; second, we consider the peak period (two hours in the morning (7-9) and two hours in the afternoon (15-17) on a representative workday) and a non-peak period (the 9 hours of the day that are not the peak). We apply the same model to Stockholm and Karlstad, but the definition of the user costs will differ because Stockholm has separate bus lanes.

## 2.3 Preferences

The travelers' preferences are represented by a quasi-linear quadratic utility function  $U$ . Utility is the sum of utility derived from non-transport goods (money  $m$ ) and the sub-utility functions  $B^{j,k}$  for all transport goods. The superscripts  $k$  and  $j$  stand for user group and trip length ( $j$ = short [s], long [l]). The travellers of a certain user group and trip length can choose between three modes indicated by the subscripts  $h$  ( $h$  = car [c], bus[b], bicycle [v]), and between the peak (p) and the off-peak (o) periods. We assume that the demand for short and long trips is independent. Short trips are below 4 km and long trips are above 4 km.

Utility function for a representative user in user group  $k$  is given by

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<sup>4</sup> This is common in denser cities where it is difficult to find space for the bicycle lane along the bus stop.

$$U^k = m^k + B^{s,k}(q_{c,p}^{s,k}, q_{b,p}^{s,k} q_{v,p}^{s,k}, q_{c,o}^{s,k}, q_{b,o}^{s,k} q_{v,o}^{s,k}) + B^{l,k}(q_{c,p}^{l,k}, q_{b,p}^{l,k} q_{v,p}^{l,k}, q_{c,o}^{l,k}, q_{b,o}^{l,k} q_{v,o}^{l,k}), \quad (0)$$

where  $m$  is the utility from goods other than transportation. The quasi-linear function removes the income effect, justified by the limited share of the household costs allocated to transport.

The quadratic sub-utility function for short and long trips has the form

$$\begin{aligned} B^{j,k}(q_{c,p}^{j,k}, q_{b,p}^{j,k} q_{v,p}^{j,k}, q_{c,o}^{j,k}, q_{b,o}^{j,k} q_{v,o}^{j,k}) = & a_{c,p}^{j,k} q_{c,p}^{j,k} + a_{b,p}^{j,k} q_{b,p}^{j,k} + a_{v,p}^{j,k} q_{v,p}^{j,k} + a_{c,o}^{j,k} q_{c,o}^{j,k} + a_{b,o}^{j,k} q_{b,o}^{j,k} + \\ & a_{v,o}^{j,k} q_{v,o}^{j,k} - 0.5 b_{c,p}^{j,k} (q_{c,p}^{j,k})^2 - 0.5 b_{b,p}^{j,k} (q_{b,p}^{j,k})^2 - 0.5 b_{v,p}^{j,k} (q_{v,p}^{j,k})^2 - 0.5 b_{c,o}^{j,k} (q_{c,o}^{j,k})^2 - \\ & 0.5 b_{b,o}^{j,k} (q_{b,o}^{j,k})^2 - 0.5 b_{v,o}^{j,k} (q_{v,o}^{j,k})^2 - i_{cp,bp}^{j,k} q_{cp}^{j,k} q_{bp}^{j,k} - i_{cp,co}^{j,k} q_{cp}^{j,k} q_{co}^{j,k} - i_{co,bo}^{j,k} q_{co}^{j,k} q_{bo}^{j,k} - \\ & i_{bp,bo}^{j,k} q_{bp}^{j,k} q_{bo}^{j,k} - i_{bp,vp}^{j,k} q_{bp}^{j,k} q_{vp}^{j,k} - i_{bo,vo}^{j,k} q_{bo}^{j,k} q_{vo}^{j,k}, \end{aligned} \quad (0)$$

where  $q$  is the number of trips by the representative individual. The parameters  $i$ ,  $a$  and  $b$  are the parameters in the sub-utility function  $B$ .

By definition, the willingness to pay equals the user cost (including tolls and fares) in equilibrium.

## 2.4 User cost

We define the user cost by mode, excluding the tolls and fares. The subscripts representing the peak and off-peak periods ( $p, o$ ) are omitted in this section, as the equations are applicable to both periods. The user cost for cyclists is

$$uc_v^{j,k} = d^j c_v + d^j W_v^k t_v, \quad (0)$$

where  $c_v$  is the fixed pecuniary distance cost, and  $d^j$  is travel distance,  $W_v^k$  is the value of time for cycling,  $t_v$  is the travel time per distance unit

$$t_v = \alpha_v + \beta_v \frac{q_v^l + \lambda q_v^s}{n} + (1 - \varphi) \cdot \gamma_v n_s P_b P_c, \quad (0)$$

where  $\alpha_v$  is the free-flow cycling time. The second term represents the effect of congestion, depending on the number of bicycles.  $\beta_v$  is the congestion parameter and the numerator gives the total number of cyclists during the period with  $q_v^l$  denoting the number of long trips,  $q_v^s$

for short trips, and  $\lambda$  as the relative distance of short trips to long trips. The parameter  $n$  is the number of hours in the period. The travel time does not depend on the number of cars or buses because we assume that there is always a bicycle lane (between the bus stops). The third term is the additional travel time caused by the interactions with other modes at the bus stop in case the bicycle lane disappears there. It is zero when the cycling lane continues along the bus stop ( $\phi = 1$ ) and positive if it disappears at the bus stop ( $\phi = 0$ ). This additional travel time is only imposed when there is a car in the car lane and a bus at the bus stop at the same time, forcing the cyclist to wait.  $n_s$  is the average number of bus stops per km,  $P_b$  is the probability that there is a bus at the bus stop,  $P_c$  is the probability that there is a car at the bus stop, and  $\gamma_v$  is the interaction parameter.

The user cost for drivers is

$$uc_c^{j,k} = d^j c_c + d^j W_c^{inv,k} \left[ \alpha_c + \beta_c \frac{\phi \cdot (\sigma(V_b) f \cdot n) + q_c^l + \lambda q_c^s}{n \text{ cap}} + (1 - \phi)(1 - \phi) \gamma_c n_s P_b P_v + \phi \phi \gamma_c n_s P_b \right], \quad (0)$$

where  $c_c$  is the pecuniary distance cost and the second term is the time cost. The first term in the brackets,  $\alpha_c$ , is the free flow travel time per distance unit. The second term represents congestion. It depends on the number of cars, but also on whether there is a separate bus lane. The dummy variable  $\phi = 0$  if there is a separate bus lane and 1 otherwise. If there are no separate bus lanes the congestion depends on the bus frequency  $f$  (number of buses per hour) and the vehicle equivalent of a bus ( $\sigma$ ), which is a function of the number of seats on the buses  $V_b$ . The parameter  $cap$  is the number of car lanes (which are also used by buses if there is no separate bus lane). The final two terms represent the delay caused by interaction with other modes at bus stops. The first of them is zero if the bicycle lane continues along the bus stop ( $\phi = 1$ ) but not if it disappears at the bus stop ( $\phi = 0$ ) (and vice versa for the final term). The terms depend on the  $P_b$  and, if  $\phi = 1$ ; also on the probability that there is a cyclist at the bus stop  $P_v$  and the

interaction parameter  $\gamma_c$ . This is because cars are delayed only when there is a bus at the bus stop, so the passing cyclists must divert into the car lane, imposing a cost on drivers.<sup>5</sup>

The user cost for bus passengers includes the cost of the waiting and schedule delay time at the bus stop, the access cost to the bus stop, and the cost of in-vehicle time,  $t_b^{inv}$ ,

$$uc_b^{j,k} = W_b^{w,k} \frac{1}{2f} + W_b^{ac,k} \frac{L}{2n_s v_w} + d^j W_b^{inv,k} t_b^{inv} D. \quad (0)$$

The value of time is different for waiting and schedule delay time at the bus stop  $W_b^{w,k}$ , access time  $W_b^{ac,k}$  and in-vehicle time  $d^j W_b^{inv,k}$ . The waiting time at the bus stop is derived based on the assumption that the bus stops and the passengers are distributed uniformly along the bus route.<sup>6</sup> It equals average walking distance (length of the bus route  $L$  divided by  $2n_s$ ) divided by the walking speed  $v_w$ . The in-vehicle time for bus users is augmented by a discomfort multiplier  $D$  depending on the occupancy of the bus (see (14)). The in-vehicle time is

$$t_b^{inv} = \alpha_b + \beta_b \frac{\sigma(V_b)f \cdot n + \phi \cdot (q_c^l + \lambda q_c^s)}{n(1-\phi) + cap \phi n} + t^s + (1 - \phi) \cdot \gamma_b n_s P_v, \quad (0)$$

where  $\alpha_b$  is the free-flow bus travel time. The second term is the additional travel time due to congestion. It depends on the bus frequency and the vehicle equivalent of a bus  $\sigma$ , which is a function of the number of seats on the buses  $V_b$ . It also depends on the number of cars, if there is no separate bus lane indicated by  $\phi$ . The third term is the time spent by buses at bus stops,  $t^s$ , which is defined as

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<sup>5</sup> The case where there is no separate bus lane,  $\phi=1$ , and the cycling lane disappears at the bus stop,  $\phi=0$ , is neglected in this model, since we do not have this case in Stockholm or in Karlstad.

<sup>6</sup> It is reasonable to assume that bus passengers arrive randomly and do not plan their trip according to the time table when the headway is short, as in Stockholm. In Karlstad, however, where the headway is around 15 minutes or more it is more likely that people plan when to arrive at the bus stop. Because of this, the value of headway when the headways is 11-30 minutes is 80% of the value of headway when the headway < 10 min according to the Swedish CBA guidelines. For this reason, we assume that the value of headway in Karlstad is 80% of the headway in Stockholm. It is also reasonable to assume that the population is distributed uniformly along the bus route in an inner city.

$$t^s = n_s[s + \zeta(q_b^l + q_b^s)/(fn - n_s)]. \quad (0)$$

It includes a fixed time cost  $s$  (per stop) for the bus entering and leaving the stop, and a variable cost depending on the number of passengers boarding and alighting at each stop.  $\zeta$  is the time needed for a passenger to board and alight, which is then multiplied by the average number of passengers who board and alight per stop.

The final term of (12) reflects the delay arising from interaction with bicycles at the bus stop, where  $\gamma_b$  is an interaction parameter. The time delay at bus stops (last term in (12)) does not depend on the number of cars even if there is no bus lane, because we assume that the bus has priority.

The discomfort/crowding multiplier,

$$D = p^{n_{on}/V_b}, \quad (0)$$

is equal to one when the bus is empty, and is equal to  $p$  when the number of passengers on board  $n_{on}$  reaches the seating capacity of bus  $V_b$ , and continues to increase afterwards.<sup>7</sup> The number of passengers on board at a certain point of time is

$$n_{on} = r_{on} \cdot (q_b^l + \lambda q_b^s)/(fn - ), \quad (0)$$

which is given by an on-board ratio  $r_{on}$  multiplied by the average number of passenger trips per bus. Since not all bus passengers travel the full route  $r_{on} < 1$ . The parameter  $r_{on}$  is lower than 1 because not all of the long and short trips have maximum lengths ( $s$  and  $l$ ). For simplicity we assume that  $r_{on}$  is equal for long and short trips.

Finally, the probability that a car, a bicycle and bus is at a bus stop is

$$\begin{aligned} P_b &= \max[t^s \cdot f/60, 1] \\ P_c &= \max[TS_c \cdot (q_c^l + \lambda q_c^s)/(60 \cdot n - ), 1] \\ P_v &= \max[TS_v \cdot (q_v^l + \lambda q_v^s)/(60 \cdot n - ), 1] \end{aligned} \quad ,(0)$$

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<sup>7</sup> We use a continuous function, inducing a small crowding effect even with few passengers because a step function makes it difficult for the model to converge. Moreover, the user cost increases with the number of bus passengers also due to longer time for boarding and alighting.

where  $TS_c$  and  $TS_v$  are the times it takes for bicycles and cars to pass a bus stop. The probabilities take the maximum value of one when the time that all buses together spend at the bus stop exceeds the length of the period.

## 2.5 Cost of public transport supply

The supply cost of buses is

$$C_b = n_s \cdot c_s + n_p \cdot f_p \cdot k_p + n_o \cdot f_o \cdot k_o, \quad (0)$$

where  $c_s$  is the capital and maintenance cost of one bus stop, and  $k_p$  and  $k_o$  are the operation cost per bus service in peak and off-peak periods, respectively (hence this is the cost per kilometre times the distance of the bus route).

## 2.6 External costs

The external cost (other than congestion) per kilometre for cars and buses ( $e_c, e_b$  respectively) is the sum of the external cost of noise ( $e_{noise}$ ), the external cost of accidents ( $e_{accident}$ ), the external cost of health damaging and climate gas emissions (particles, NOX and CO<sub>2</sub>) ( $e_{c\_emission}$ ), minus the external cost that is partly internalized by fuel taxes ( $e_{internal}$ ).

The external costs are different for Stockholm and Karlstad, see section 3.

We assume that the external cost of cycling (except for congestion) is zero (Niska and Eriksson, 2013).

## 2.7 The welfare optimum

Our welfare function,  $\Omega$ , equals the user surplus and the producer surplus minus total external cost other than congestion (we assume here that the taxes collected to finance possible deficits for the producer are non-distortive, but we change this assumption in section 4.2)

$$\Omega = \sum_{j,k} B^{j,k}(\dots, q_{h,i}^{j,k}, \dots) - \sum_{j,k,h,i} q_{h,i}^{j,k} uc_{h,i}^{j,k} - C_b - e_{c,i}^l q_{c,i}^l - e_{c,i}^s q_{c,i}^s - e_{b,i} f_i n_i. \quad (0)$$

We eliminate the bus fare from the user cost and revenues from the producer surplus since this is only a transfer. In user equilibrium the user cost (including fares and tolls  $\tau$ ) equals the willingness to pay

$$uc_{h,i}^{j,k} + \tau_{h,i}^j = \frac{dB^{j,k}}{dq_{h,i}^{j,k}}, \forall j, k, h, i. \quad (0)$$

We derive the optimal tolls by differentiating the welfare function with respect to demand for the three modes and trip lengths, setting each derivative to zero, and by plugging in the user equilibrium condition, we get the optimal fare or toll by trip length

$$\begin{aligned} \tau_{v,i}^j &= \sum_h \sum_k \sum_j q_{h,i}^{j,k} \frac{\partial uc_{h,i}^{j,k}}{\partial q_{v,i}^j} \\ \tau_{b,i}^j &= \sum_h \sum_k \sum_j q_{h,i}^{j,k} \frac{\partial uc_{h,i}^{j,k}}{\partial q_{b,i}^j} \\ \tau_{c,i}^j &= \sum_h \sum_k \sum_j q_{h,i}^{j,k} \frac{\partial uc_{h,i}^{j,k}}{\partial q_{c,i}^j} + e_{c,i}^j \end{aligned} \quad .(0)$$

These equations demonstrate that the optimal toll or fare of a trip equals the marginal external costs. In our model we have a complex interaction between modes which influence the externalities and therefore are reflected in the pricing structure. For instance, the optimal toll for a car trip is the sum of the congestion cost accruing to other drivers, the delay for cyclists at bus stops, and other externalities. The effect on the user cost of the delayed drivers and cyclists depends on their values of time (depending on the user groups) and the trip length. Therefore, the external costs are summed across user groups ( $k = 1, 2, 3$ ) and trip length  $j = s, l$ .

One can also derive optimal bus frequency and bus stop spacing analytically, given the optimal tolls. But we want to focus on the essentials. The bus frequency and bus stop spacing are optimized numerically, based on the objective function (18) and we present the results in Section 4.

### 3 STUDY AREA AND MODEL CALIBRATION

This section describes how the model is calibrated for a representative bus corridor in the centre of Karlstad, located in West Sweden with a population of slightly more than 60 000 inhabitants, and for the busiest bus line (No. 4) in central Stockholm. There are 8 bus lines going through the city centre of Karlstad and the representative bus line we model is the average of them (see appendix A). The model calibration for Stockholm is similar to the one in Börjesson et al. (2018). The baseline scenario is calibrated for the year 2014.

#### 3.1 Study Area

The two bus corridors under study are described in Table 2. We consider four peak hours and nine off-peak hours for a representative day. During the four peak hours, the Stockholm corridor has over 40 times as many passengers as Karlstad has. The full length of the bus lines is similar but the bus travel time is longer in Stockholm, due to more crowding and longer stops for boarding and alighting. The peak frequency is three times higher in Stockholm: 12 versus 4 buses per hour. While the off-peak frequency of buses in Stockholm is very close to the peak frequency (11 and 12), off-peak frequency in Karlstad is only half of the peak frequency (2 and 4). However, the number of bus stops per km and the fare are surprisingly similar in the two cities. There are no national regulations regarding fare or bus stop spacing, but there are conventions and rules of thumb.

Table 2 also includes the baseline monetary travel costs (tolls, fares and operation costs of bus, car and bicycle), and baseline travel times. The bus travel times are taken from the official timetables and the car travel times are from Google Maps. To compute the cycling travel time, we assume a speed of 12 km/h in the peak (and in the off-peak).

Table 2: The bus corridors in the peak (p) and the off-peak (o).

| Corridor                           | Karlstad (p) | (o)  | Stockholm (p) | (o)  |
|------------------------------------|--------------|------|---------------|------|
| Frequency(#/h) ( $n_h$ ) base case | 4            | 2    | 12            | 11   |
| #stops/km ( $n_s$ ) base case      | 2.51         | 2.51 | 2.58          | 2.58 |

|                                                               |        |        |        |        |
|---------------------------------------------------------------|--------|--------|--------|--------|
| Bus trip distance full route one way $L$ (km)                 | 13     | 13     | 12     | 12     |
| Bus trip time full route one way (min)                        | 38     | 33     | 50     | 44     |
| # passengers per h and direction                              | 181    | 47     | 4063   | 1806   |
| On board ratio for long and short trips $r_{on}$ <sup>8</sup> | 0.65   | 0.65   | 0.22   | 0.22   |
| Seats in the bus ( $V_b$ )                                    | 45     | 45     | 84     | 84     |
| Operation cost per bus km (€)                                 | 6.1    | 2.6    | 8.4    | 3.3    |
| Marginal cost of bus stop per day (€)                         | 5.7    | 5.7    | 5.7    | 5.7    |
| Fare (long and short trips) (€)                               | 2      | 2      | 2      | 2      |
| Car travel time full route one way (min)                      | 21     | 17     | 70     | 43     |
| Car toll (long and short trips) (€)                           | 0      | 0      | 1.8    | 1      |
| User cost per car km (€)                                      | 0.18   | 0.18   | 0.18   | 0.18   |
| Cycling travel time full route one way                        | 65     | 65     | 60     | 60     |
| Cycling toll (long and short trips) (€)                       | 0      | 0      | 0      | 0      |
| User cost per cycling km (€)                                  | 0.0008 | 0.0008 | 0.0008 | 0.0008 |

Bus is the only public transport mode that is available in the two corridors. The number of peak and off-peak trips per day by mode, and mode shares, by user group are given in Table 3. In Karlstad, the share of bus trips is lower than in Stockholm, and the shares of car and cycling trips are higher. The bus traffic volumes are taken from traffic count data, combined with shares in travel surveys to compute the number of trips by user group and trip lengths for all other modes and user groups.<sup>9</sup>

For Stockholm the user groups correspond to income groups. In the Karlstad travel survey, however, income is not available and we use occupation groups for which income differ: Employed (high income), Retired and others<sup>10</sup> (medium-high income) and Students (low income). For simplification we will refer to the occupation groups as income groups in the rest of the paper. The income level of the user groups comes from national statistics, see Appendix C. Table 3 also shows the in-vehicle value of time (VTT) for bus by user group. This is

<sup>8</sup> From passenger counts on the buses.

<sup>9</sup> The surveys in Karlstad and in Stockholm are both postal surveys conducted from a random sample of 6500 citizens of Karlstad and 30 000 in Stockholm, between 16 and 84 years old. The Karlstad survey was conducted in 2014 and the Stockholm survey in 2006. The traffic counts for the bus services are obtained from the public transport operators. The traffic counts for car are obtained from the municipalities.

<sup>10</sup> Unemployed, parental leave and sick leave.

calculated using the Swedish national value of time study, applying an income elasticity of 0.7 (Börjesson and Eliasson, 2014). For the mid income group the VTT for waiting time is 12.2 €/h for Stockholm, 9.8 €/h for Karlstad<sup>11</sup>, and for access time 10.2 €/h.

Table 3: % of trips made by a certain group in different modes according to surveys, and bus in-vehicle VTT, by user group.

|                  | Car  | Bus  | Bicycle | Total | VTT<br>Bus in-<br>vehicle<br>€/h | Population |
|------------------|------|------|---------|-------|----------------------------------|------------|
| <i>Karlstad</i>  |      |      |         |       |                                  |            |
| Employed (p)     | 24.6 | 3.1  | 15.1    | 42.8  | 9.2                              | 3664       |
| (o)              | 13.9 | 1.8  | 7.7     | 23.4  |                                  |            |
| Retired (p)      | 6.7  | 0.6  | 1.6     | 8.8   | 7.2                              | 1638       |
| (o)              | 2.1  | 0.4  | 0.7     | 3.2   |                                  |            |
| Students (p)     | 4.0  | 5.8  | 3.5     | 13.4  | 6.0                              | 1402       |
| (o)              | 2.0  | 3.4  | 3.0     | 8.4   |                                  |            |
| Total (p)        | 35.3 | 9.5  | 20.2    | 65.0  |                                  | 6703       |
| (o)              | 18.0 | 5.6  | 11.4    | 35.0  |                                  |            |
| <i>Stockholm</i> |      |      |         |       |                                  |            |
| High-income (p)  | 3.4  | 9.3  | 4.0     | 16.6  | 9.7                              | 22407      |
| (o)              | 3.4  | 9.3  | 4.0     | 16.6  |                                  |            |
| Mid-income (p)   | 4.2  | 12.9 | 4.8     | 21.9  | 7.2                              | 29804      |
| (o)              | 4.2  | 12.9 | 4.8     | 21.9  |                                  |            |
| Low-income(p)    | 0.9  | 8.5  | 2.0     | 11.5  | 4.4                              | 23464      |
| (o)              | 0.9  | 8.5  | 2.0     | 11.5  |                                  |            |
| Total (p)        | 8.5  | 30.7 | 10.8    | 50.0  |                                  | 75675      |
| (o)              | 8.5  | 30.7 | 10.8    | 50.0  |                                  |            |

The operation costs for buses are computed based on the capital cost and cost of drivers, fuel, and maintenance. Details can be found in Appendix B. The operation cost per vehicle kilometre is higher in Stockholm because of the lower speed (impacting capital cost, fuel cost and the cost for drivers). Appendix B also shows how the cost for bus stops is computed.

<sup>11</sup> As suggested by one of the referees, waiting time for less frequent bus service should be lower because passengers tend to schedule their arrivals at bus stops according to the timetable. Therefore, the VTT for waiting in Karlstad is adjusted downward by 20%.

The external costs for road traffic are higher in the inner city of Stockholm than in Karlstad. Based on the components of external costs in Table 4, the external costs for cars in Karlstad and Stockholm, respectively, are 0.036 Euro/vehicle km and 0.048 Euro/vehicle km. For buses the external cost is 0.122 Euro/vehicle km for city buses in Karlstad and 0.176 Euro/vehicle km for city buses in Stockholm.

A key difference between the cities is that in the Stockholm corridor there is a separate bus lane ( $\phi=0$ ), whereas the Karlstad corridor has none ( $\phi=1$ ). Moreover, in Stockholm the bicycle lane disappears along the bus stop ( $\varphi=0$ ), whereas this is not the case in Karlstad ( $\varphi=1$ ).

### 3.2 Calibration

We calibrate<sup>12</sup> the reference parameters  $a$ ,  $b$  and  $i$  in our model (see (7)) to the baseline situation for which we use a number of data sources: a) observed trip quantities by mode and user group, baseline money costs and time by travel time component; b) the values of time for all travel time components by user group; c) the price and cross-price elasticities (Table 4 and Appendix D); and d) speed-flow functions (Appendix E).

Cross-price elasticities between bus and car trips, and own elasticities for car use are computed for Stockholm using data from the introduction of congestion charges (Börjesson et al., 2017). For public transport we assume an own-price elasticity of -0.4. This is found in many studies (Litman, 2004) and in the national Swedish transport model. The cross-time elasticity for number of cycling trips, with respect to bus in-vehicle travel time of a bus trip, is taken from the national transport model. The cross-price elasticities are adjusted according to the mode shares in Karlstad and in Stockholm and are consistent with the suggested diversion factors in RAND (2018).

For all user groups and trip distances, the price elasticities are converted to generalized price elasticities by multiplying the ratios of full user cost of a trip to money cost of a trip. As a

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<sup>12</sup> Appendix D reports all the elasticities of different user groups and the details of the calibration.

result, we have higher generalized price elasticities for longer trips and for high income groups. Similarly, the cross-time elasticity for cycling is converted to generalized price elasticities by multiplying the ratios of total user cost of a trip to time cost of the trip.

Due to space limit, the parameters of all the speed-flow relationships ( $\alpha$  and  $\beta$ ) for buses, cars and cycling, and other related parameters are reported in Appendix E.

Table 4: Price elasticities with respect to money cost and external costs. The external costs are taken from national guidelines.<sup>13</sup> See Appendix D for all elasticities and Appendix E for other parameters.

| Parameter                                    | Notation          | Karlstad | Stockholm |
|----------------------------------------------|-------------------|----------|-----------|
| Own elasticity, bus, peak and off-peak       | $\varepsilon_b$   | -0.4     | -0.4      |
| Own elasticity, car, peak and off-peak       | $\varepsilon_c$   | -0.54    | -0.54     |
| Own elasticity, bicycle, peak and off-peak   | $\varepsilon_v$   | -0.8     | -0.8      |
| Externalities, car, noise (€/veh km)         | $e_{c\_noise}$    | 0.020    | 0.020     |
| Externalities, car, accidents (€/veh km)     | $e_{c\_accident}$ | 0.024    | 0.024     |
| Externalities, car, emissions (€/veh km)     | $e_{c\_emission}$ | 0.036    | 0.048     |
| Externalities intern. in fuel tax (€/veh km) | $e_{c\_internal}$ | -0.044   | -0.044    |
| Exter. not intern. in fuel tax (€/veh km)    | $e_c$             | 0.036    | 0.048     |
| Externalities, bus, noise (€/veh km)         | $e_{b\_noise}$    | 0.100    | 0.1       |
| Externalities, bus, accidents (€/veh km)     | $e_{b\_accident}$ | 0.058    | 0.058     |
| Externalities, bus, emissions (€/veh km)     | $e_{b\_emission}$ | 0.162    | 0.216     |
| Externalities intern. in fuel tax (€/veh km) | $e_{b\_internal}$ | -0.198   | -0.198    |
| Exter. not intern. in fuel tax (€/veh km)    | $e_b$             | 0.122    | 0.176     |

## 4 RESULTS

In section 4.1, we optimize a set of transport policy instruments one by one: pricing of all travel modes (cars, bus, bicycle), bus frequency, and number of bus stops, and compare the results for Karlstad and Stockholm. We also optimize different combinations of the policies, such that we have five policy scenarios in total. In section 4.2 we explore how the optimal policies

<sup>13</sup> *Method of analysis and economic values for the transport sector, ASEK*, prepared by the Swedish Transport Administration (Trafikverket).

change when assuming distortive taxes to fund deficits (marginal cost of public funds  $>1$ ). In Section 4.3 we briefly explore the distribution effect of policy changes across income groups.

## 4.1 Optimal policy

We define five scenarios: (1) optimizing all three policy variables (prices, bus frequency, and number of bus stops); (2) optimize all prices; (3) bus frequency and number of bus stops, (4) zero tolls; (5) alternative arrangement of bus lane.

Table 5 reports all the policy variables except prices for all five scenarios, for Karlstad and Stockholm, respectively. Table 6 reports the optimal prices. Table 7 reports the effects of policy changes on the number of trips. We first outline the key results for the two cities, using results from Tables 5, 6 and 7 and focus then on the implications for the bus subsidies.

### 4.1.1 Overall findings

Focusing first on Karlstad, making bus use cheaper in the off peak and introducing tolls for car use is the main policy advice. In optimum the bus fares in the peak period are much lower than the fares in the baseline, and the off-peak bus fares are even lower. The optimal frequency is slightly lower than the baseline in peak (3 rather than 4), but close to the baseline in the off-peak. The tolls for cars and cycling in Karlstad are low compared to Stockholm, because the marginal external congestion cost is lower, but the optimal car toll is positive and thus larger than in the baseline.

In Stockholm, the story is different. Stockholm has already a high modal bus share and the most welfare-improving policy intervention is to decrease bus fares by more than 75% in the off peak and by 60% in the peak period. This is accompanied by higher frequencies mainly in the peak period.<sup>14</sup> The optimal car tolls are much larger than the current for the long trips in the peak. This is in line with the current cordon toll that does not differentiate between short

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<sup>14</sup> We assume one can fit 21 buses/hour on this line. If for engineering reasons, one can not load/unload the passengers sufficiently quickly, one may need to limit the frequency and this would give higher peak bus fares.

and long trips. In optimum the total number of trips increases, as the increase in bus trips dominates the decrease in car trips.

In Stockholm the share of cyclists decreases if the fares and frequencies of the bus are optimized. In Karlstad, however, the number of cyclists does not change a lot when prices are optimized.

The complementary scenarios (2,3,4 and 5) help to understand the relative importance of the policy reforms in the global optimal scenario 1. In Karlstad, changing the baseline fares and tolls to the optimal increases the welfare more than optimizing frequency. This is because the current bus fares and tolls are further from the optimum than the frequency. In Stockholm, however, adjusting the price reform and the frequency reform to the optimal increases welfare to the same extent. In Stockholm, the frequencies need to be increased in both peak and off peak, even when prices are kept at baseline level (cfr; scenario 3).

Comparing the results for Karlstad with the Stockholm corridor, we find that the optimal bus stop spacing is denser in Karlstad than in Stockholm (1.9/km vs 2.2/km). The reason is that the fixed stop time (independent from the number of travelers boarding and alighting) is relatively more important in Karlstad because there are fewer passengers boarding at the bus stop. Hence, because the bus stop time per boarding and alighting traveler is longer in Karlstad, the disutility for travelers to walk longer to the bus stop is relatively smaller than the disutility of longer travel time for the passengers on the bus.

The fourth and fifth scenario are constructed under the constraints that road tolls (and cycling tolls) cannot be implemented. These scenarios increase the comparability of our study with other cities where road pricing has not been implemented. Three results stand out. First there is a welfare loss. Second, it still pays off to reform bus prices: lower peak and off peak-prices in Stockholm and Karlstad. Third, the introduction of a bus lane in Karlstad has a small welfare

cost (comparing scenario 5 and 4). But in Stockholm, where there is already a bus lane, taking away the bus lane would decrease welfare compared to the baseline.

#### 4.1.2 What subsidies are needed for buses?

We now focus on the bus subsidies needed in the two cities when pricing and frequency policies can be optimized. The subsidies needed are reported as a % of the total supply costs in column 5 of table 5. Column 4 reports the total supply cost per day.

The main finding is that in all Karlstad scenarios the optimal subsidy rate is large (83% in the full optimum scenario 1) but in Stockholm, only little subsidy (6%) is needed in the full optimum scenario 1; there is even a surplus in scenarios 2 and 3.

This confirms our intuition developed in section 2 that once demand density is large enough and translates into a high frequency, the positive waiting time externality becomes small and is compensated by the negative crowding and road congestion externality.

This result does stand out even if car tolls are not allowed: only a 23% subsidy is required, this is a lower rate than in any of the scenarios in Karlstad.

Table 5: Baseline and policy scenario. (p: peak, o: off-peak)

| Policy scenario               | Welfare gain<br>€/day | # stops/km | Frequency<br>/hour | Supply cost<br>€/day | Subsidy % of supply cost | # bus trips<br>/day |
|-------------------------------|-----------------------|------------|--------------------|----------------------|--------------------------|---------------------|
| <b>Karlstad</b>               |                       |            |                    |                      |                          |                     |
| Baseline (p)                  | -                     | 2.54       | 4.00               | 7704                 | 41                       | 1442                |
| (o)                           |                       |            | 2.00               |                      |                          | 845                 |
| 1. All prices +freq +stop (p) | 2038                  | 1.89       | 2.93               | 6621                 | 83                       | 1767                |
| (o)                           |                       |            | 2.26               |                      |                          | 1223                |
| 2. All prices only (p)        | 1579                  | 2.54       | 4.00               | 7704                 | 87                       | 1943                |
| (o)                           |                       |            | 2.00               |                      |                          | 1167                |
| 3. Frequency +stops (p)       | 511                   | 1.90       | 2.83               | 6475                 | 33                       | 1283                |
| (o)                           |                       |            | 2.24               |                      |                          | 894                 |
| 4. = 1 + zero tolls (p)       | 1248                  | 1.89       | 2.92               | 6605                 | 91                       | 1751                |
| (o)                           |                       |            | 2.26               |                      |                          | 1222                |
| 5. =1 + bus lane + zero tolls | 468                   | 1.87       | 3.10               | 6867                 | 91                       | 1930                |
| (o)                           |                       |            | 2.29               |                      |                          | 1247                |
| <b>Stockholm</b>              |                       |            |                    |                      |                          |                     |

|                                    |        |      |       |       |      |       |
|------------------------------------|--------|------|-------|-------|------|-------|
| Baseline (p)                       | -      | 2.58 | 12.00 | 35164 | -270 | 32500 |
| (o)                                |        |      | 11.00 |       |      | 32500 |
| 1. All prices +freq +stops (p)     | 30169  | 2.21 | 21.68 | 59604 | 6    | 43367 |
| (o)                                |        |      | 17.23 |       |      | 43667 |
| 2. All prices only (p)             | 10824  | 2.58 | 12.00 | 35164 | -103 | 36972 |
| (o)                                |        |      | 11.00 |       |      | 40336 |
| 3. Frequency +stops (p)            | 14919  | 2.20 | 20.24 | 55838 | -150 | 35665 |
| (o)                                |        |      | 16.21 |       |      | 34214 |
| 4. =1 + zero tolls (p)             | 22272  | 2.20 | 21.73 | 59738 | 23   | 43476 |
| (o)                                |        |      | 17.27 |       |      | 43781 |
| 5. =1 +no bus lane +zero tolls (p) | -58614 | 2.38 | 16.33 | 49724 | 46   | 28951 |
| (o)                                |        |      | 16.33 |       |      | 38500 |

Table 6: Fares and tolls: Optimal pricing scenario. (p: peak, o: off-peak)

| Policy scenario                 | Short bus trips | Long bus trips | Short car trips | Long car trips | Short bicycle trips | Long bicycle trips |
|---------------------------------|-----------------|----------------|-----------------|----------------|---------------------|--------------------|
| <b>Karlstad</b>                 |                 |                |                 |                |                     |                    |
| Baseline (p)                    | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| (o)                             | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| 1. All prices + freq +stops (p) | 0.3             | 0.6            | 0.4             | 1.2            | 0.0                 | 0.1                |
| (o)                             | 0.1             | 0.2            | 0.2             | 0.7            | 0.0                 | 0.0                |
| 2. All prices only (p)          | 0.3             | 0.5            | 0.4             | 1.2            | 0.0                 | 0.1                |
| (o)                             | 0.1             | 0.2            | 0.2             | 0.7            | 0.0                 | 0.0                |
| 3. Frequency +stops (p)         | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| (o)                             | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| 4. =1 +zero tolls (p)           | 0.3             | 0.3            | 0.0             | 0.0            | 0.0                 | 0.0                |
| (o)                             | 0.0             | 0.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| 5. =1+bus lane + zero tolls (p) | 0.3             | 0.3            | 0.0             | 0.0            | 0.0                 | 0.0                |
| (o)                             | 0.0             | 0.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| <b>Stockholm</b>                |                 |                |                 |                |                     |                    |
| Baseline (p)                    | 2.0             | 2.0            | 1.8             | 1.8            | 0.0                 | 0.0                |
| (o)                             | 2.0             | 2.0            | 1.0             | 1.0            | 0.0                 | 0.0                |
| 1. All prices + freq +stops (p) | 0.6             | 0.8            | 1.0             | 4.0            | 0.1                 | 0.3                |
| (o)                             | 0.4             | 0.5            | 0.5             | 2.1            | 0.0                 | 0.1                |
| 2. All prices only (p)          | 1.0             | 1.3            | 1.0             | 4.1            | 0.1                 | 0.3                |

|                                    |     |     |     |     |     |     |
|------------------------------------|-----|-----|-----|-----|-----|-----|
| (o)                                | 0.6 | 0.8 | 0.5 | 2.1 | 0.0 | 0.1 |
| 3. Frequency +stops (p)            | 2.0 | 2.0 | 1.8 | 1.8 | 0.0 | 0.0 |
| (o)                                | 2.0 | 2.0 | 1.0 | 1.0 | 0.0 | 0.0 |
| 4. =1+zero tolls (p)               | 0.6 | 0.7 | 0.0 | 0.0 | 0.0 | 0.0 |
| (o)                                | 0.4 | 0.4 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5. =1 +no bus lane +zero tolls (p) | 0.5 | 0.5 | 0.0 | 0.0 | 0.0 | 0.0 |
| (o)                                | 0.3 | 0.3 | 0.0 | 0.0 | 0.0 | 0.0 |

Table 7: Number of trips (in thousands) in baseline and policy scenarios. (p: peak, o: off-peak)

| Policy scenario                 | Short bus trips | Long bus trips | All bus trips | Short car trips | Long car trips | All car trips | Short bicycle trips | Long bicycle trips | All bicycle trips |
|---------------------------------|-----------------|----------------|---------------|-----------------|----------------|---------------|---------------------|--------------------|-------------------|
| <b>Karlstad</b>                 |                 |                |               |                 |                |               |                     |                    |                   |
| Baseline (p)                    | 0.5             | 0.9            | 1.4           | 1.8             | 3.5            | 5.4           | 1.1                 | 2.0                | 3.1               |
| (o)                             | 0.3             | 0.6            | 0.8           | 0.9             | 1.8            | 2.7           | 0.6                 | 1.1                | 1.7               |
| 1. All prices + freq +stops (p) | 0.6             | 1.2            | 1.8           | 1.7             | 2.6            | 4.3           | 1.0                 | 2.0                | 3.0               |
| (o)                             | 0.4             | 0.8            | 1.2           | 0.8             | 1.3            | 2.2           | 0.6                 | 1.1                | 1.7               |
| 2. All prices only (p)          | 0.7             | 1.3            | 1.9           | 1.7             | 2.6            | 4.3           | 1.0                 | 2.0                | 3.0               |
| (o)                             | 0.4             | 0.8            | 1.2           | 0.8             | 1.3            | 2.2           | 0.6                 | 1.1                | 1.7               |
| 3. Frequency +stops (p)         | 0.4             | 0.8            | 1.3           | 1.9             | 3.6            | 5.4           | 1.1                 | 2.0                | 3.1               |
| (o)                             | 0.3             | 0.6            | 0.9           | 0.9             | 1.8            | 2.7           | 0.6                 | 1.1                | 1.7               |
| 4. =1+Zero tolls (p)            | 0.6             | 1.1            | 1.8           | 1.8             | 3.5            | 5.3           | 1.0                 | 2.0                | 3.1               |
| (o)                             | 0.4             | 0.8            | 1.2           | 0.9             | 1.7            | 2.6           | 0.6                 | 1.1                | 1.7               |
| 5.=1+ bus lane +zero tolls(p)   | 0.6             | 1.3            | 1.9           | 1.8             | 3.1            | 4.9           | 1.0                 | 2.0                | 3.0               |
| (o)                             | 0.4             | 0.8            | 1.2           | 0.9             | 1.7            | 2.5           | 0.6                 | 1.1                | 1.7               |
| <b>Stockholm</b>                |                 |                |               |                 |                |               |                     |                    |                   |
| Baseline (p)                    | 14.4            | 18.1           | 32.5          | 4.0             | 5.0            | 9.0           | 5.1                 | 6.3                | 11.4              |
| (o)                             | 14.4            | 18.1           | 32.5          | 4.0             | 5.0            | 9.0           | 5.1                 | 6.3                | 11.4              |
| 1. All prices + freq +stops (p) | 19.0            | 24.4           | 43.4          | 4.4             | 3.8            | 8.2           | 4.6                 | 5.3                | 9.9               |
| (o)                             | 19.3            | 24.4           | 43.7          | 4.3             | 3.7            | 7.9           | 4.6                 | 5.6                | 10.2              |

|                                    |      |      |      |     |     |      |     |     |      |
|------------------------------------|------|------|------|-----|-----|------|-----|-----|------|
| 2. All prices only (p)             | 17.0 | 20.0 | 37.0 | 4.4 | 3.9 | 8.3  | 4.7 | 5.7 | 10.4 |
| (o)                                | 18.2 | 22.2 | 40.3 | 4.3 | 3.8 | 8.1  | 4.7 | 5.8 | 10.5 |
| 3. Frequency +stops (p)            | 15.2 | 20.5 | 35.7 | 4.0 | 4.9 | 8.9  | 5.0 | 6.2 | 11.2 |
| (o)                                | 14.9 | 19.3 | 34.2 | 4.0 | 4.9 | 8.9  | 5.0 | 6.3 | 11.3 |
| 4. =1 +zero tolls (p)              | 19.0 | 24.5 | 43.5 | 4.7 | 5.6 | 10.3 | 4.7 | 5.9 | 10.5 |
| (o)                                | 19.3 | 24.5 | 43.8 | 4.6 | 5.6 | 10.2 | 4.7 | 5.8 | 10.5 |
| 5. =1+ no bus lane +zero tolls (p) | 16.8 | 12.2 | 29.0 | 4.3 | 7.2 | 11.5 | 4.9 | 6.9 | 11.7 |
| (o)                                | 18.7 | 19.8 | 38.5 | 4.0 | 6.4 | 10.5 | 4.7 | 6.2 | 10.9 |

## 4.2 Introducing a marginal cost of public funds (MCPF)

So far we have assumed that if the operator generates a deficit the government can cover it by non-distortive taxes. In the welfare function, one euro of extra user surplus is as important as one euro of government surplus. However, this assumption rarely holds, implying that collecting taxes induces a deadweight loss. In this subsection we take this into account by including the marginal cost of public funds (MCPF) in the welfare function.

$$\Omega = \sum_{j,k} B^{j,k}(\dots, q_h^{j,k}, \dots) - \sum_{j,k,h} q_h^{j,k} u c_h^{j,k} - C_b - e_c^l q_c^l - e_c^s q_c^s - e_b f n_h - (MCPF - 1) * (C_b - \sum_{j,k,h} q_h^{j,k} \tau_h^{j,k}). \quad (0)$$

Sørensen (2010) estimates the MCPF at 1.3 for Sweden. This means that the marginal social cost of subsidising public transport with 1 euro is 1.3 euro and generating a surplus of 1 euro in public transport generates an extra welfare gain of 0.3 euro. But this is a macro-reasoning that does not take into account that the bus and car are used for commuting trips, and this matters as the distortion on the labour market is the major component of the MCPF. Parry and Bento (2001) tell us that in that case, marginal social cost pricing is still optimal and Van Dender (2004) shows that one should ideally discriminate between commuting and non-commuting traffic, so extracting more revenue out of the non-commuters. We know that one

third of trips in Karlstad are commuting trips and we assume an identical share for Stockholm. We take into account the commuting aspect of transport pricing in the MCPF by using a value of MCPF corrected for the average share of commuting trips in the total number of trips (here 1/3). Then the corrected MCPF equals 1.2.

Accounting for the MCPF increases the optimal fares in Stockholm and in Karlstad (Appendix F). Also, the optimal frequencies are lower when MCPF increases. The optimal tolls for cycling and cars increase as well when accounting for the MCPF. The tolls and fares are higher simply because the deadweight losses when increasing the tolls and fares are smaller than the deadweight loss when increasing other taxes to raise government revenue.

The subsidy in the “all optimized” scenario in Karlstad is reduced from 83% of bus operating cost when not accounting for MCPF to 41%. Stockholm goes from a small subsidy of 6% to generating a surplus of 130%.

Table 8: Comparison of baseline and policy scenarios, Karlstad, per bus line, MCPF = 1.2.

| Policy scenario                   | Welfare gain<br>€/day | # stops/km | Frequency<br>/hour | Supply cost<br>€/day | Subsidy % of supply cost | # bus trips<br>/day |
|-----------------------------------|-----------------------|------------|--------------------|----------------------|--------------------------|---------------------|
| <b>Karlstad</b>                   |                       |            |                    |                      |                          |                     |
| Baseline (p)                      | -                     | 2.54       | 4.00               | 7704                 | 41                       | 1442                |
| (o)                               |                       |            | 2.00               |                      |                          | 845                 |
| 1. All prices +freq +stop (p)     | 3504                  | 1.87       | 2.63               | 5983                 | 41                       | 1483                |
| (o)                               |                       |            | 2.05               |                      |                          | 1018                |
| 2. All prices only (p)            | 2768                  | 2.54       | 4.00               | 7704                 | 50                       | 1690                |
| (o)                               |                       |            | 2.00               |                      |                          | 1005                |
| 3. Frequency +stops (p)           | 747                   | 1.88       | 2.62               | 6002                 | 30                       | 1239                |
| (o)                               |                       |            | 2.08               |                      |                          | 864                 |
| 4. = 1 + zero tolls (p)           | 997                   | 1.87       | 2.62               | 5966                 | 54                       | 1463                |
| (o)                               |                       |            | 2.05               |                      |                          | 1017                |
| 5. =1 + bus lane + zero tolls (p) | 211                   | 1.86       | 2.78               | 6209                 | 50                       | 1620                |
| (o)                               |                       |            | 2.08               |                      |                          | 1039                |
| <b>Stockholm</b>                  |                       |            |                    |                      |                          |                     |
| Baseline (p)                      | -                     | 2.58       | 12.00              | 35164                | -270                     | 32500               |
| (o)                               |                       |            | 11.00              |                      |                          | 32500               |
| 1. All prices +freq +stops (p)    | 22490                 | 2.21       | 19.40              | 53725                | -130                     | 37359               |
| (o)                               |                       |            | 15.68              |                      |                          | 37580               |

|                                   |        |      |       |       |      |       |
|-----------------------------------|--------|------|-------|-------|------|-------|
| 2. All prices only (p)            | 8915   | 2.58 | 12.00 | 35164 | -249 | 33042 |
| (o)                               |        |      | 11.00 |       |      | 35405 |
| 3. Frequency +stops (p)           | 12911  | 2.20 | 19.13 | 52911 | -163 | 35415 |
| (o)                               |        |      | 15.41 |       |      | 34044 |
| 4. =1 +zero tolls (p)             | 3856   | 2.20 | 19.45 | 53846 | -107 | 37443 |
| (o)                               |        |      | 15.71 |       |      | 37677 |
| 5. =1+no bus lane + zero tolls(p) | -75589 | 2.38 | 14.58 | 44399 | -50  | 24535 |
| (o)                               |        |      | 14.58 |       |      | 33025 |

### 4.3 Distributional impact

Previously, we implicitly assumed that all income groups carry the same weight in our welfare function: a gain of one Euro for a rich person counted as much as one Euro for a low-income person. That is, we only care about the sum of welfare of different income groups, but not about equity. An often-raised argument in the public debate for subsidizing public transport is that it is needed to support the low-income group. Therefore, distributional impacts are relevant.

We explore how the welfare impact of a change in bus subsidies is distributed among income groups. We take the number of trips in the baseline scenario as given and neglect the behavioural effects of a marginal change in the subsidies. Moreover, we assume that the bus subsidies are financed through a head tax, meaning that the burden of the bus subsidies is shared equally among the citizens of all income groups<sup>15</sup>.

Since higher subsidies are often argued to benefit low income citizens, we first explore how an increase in the subsidies accrues to different user groups, keeping other factors constant. This will depend on how the bus trip frequency differs between users. Table 9 shows a simple calculation of the gain/loss per citizen by income groups of a fare reduction of 0.1 euro, assuming that the decrease in bus revenue is financed by a head tax.

Table 9: How a marginal change in bus fares affects different income groups.

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<sup>15</sup> We do not propose to finance changes in the deficit in this way. We use the head tax assumption merely to obtain transparent results. De facto variations in income taxes will be the major way to finance the deficits.

|                            | high-income | mid-income | low-income |
|----------------------------|-------------|------------|------------|
| <b>Karlstad</b>            |             |            |            |
| bus trips                  | 748         | 140        | 1400       |
| Population                 | 3664        | 1638       | 1402       |
| bus trips per citizen      | 0.20        | 0.09       | 1.00       |
| subsidy per citizen        | 0.02        | 0.01       | 0.10       |
| head tax per citizen       | 0.03        | 0.03       | 0.03       |
| (Subsidy-head tax)/citizen | -0.01       | -0.03      | 0.07       |
| <b>Stockholm</b>           |             |            |            |
| bus trips                  | 19570       | 27346      | 18084      |
| Population                 | 22407       | 29804      | 23464      |
| bus trips per citizen      | 0.87        | 0.92       | 0.77       |
| subsidy per citizen        | 0.09        | 0.09       | 0.08       |
| head tax per citizen       | 0.09        | 0.09       | 0.09       |
| (Subsidy-head tax)/citizen | 0.00        | 0.01       | -0.01      |

Considering Karlstad first, we can see that the trip frequency per citizen differs hugely among income groups. The low-income group has a much higher trip frequency than the mid-income (>10 times) and high-income groups (~5 times), so that the low-income group receives more subsidies per citizen (0.10) than the mid-income (0.01) and the high-income groups (0.02). Therefore, if we consider the subsidy minus head tax per citizen (transfers between income groups), the low-income group gains while the mid- and high-income groups lose. For Stockholm, the bus trip frequency is similar among income groups. As a result, the net transfer between income groups is negligible. Hence, increasing bus subsidy might be a good redistribution policy in Karlstad but not in Stockholm. This is confirmed by a more detailed weighted welfare function analysis of the distributional effects in Appendix G.

#### 4.4 Sensitivity analysis

The cross-elasticities between the modes are in general context specific and therefore uncertain parameters. In addition, one could doubt that the low optimal bus fares, thus high subsidy rates in Karlstad could be driven merely by the high diversion factors from the car mode to the bus mode. For these two reasons, we checked the sensitivity of our results by doubling the diversion

ratio from the bus mode to the car mode. We also test with a higher discomfort coefficient. With these alternative parameter values, the main conclusions appear to be robust<sup>16</sup>.

## 5 POLICY CONCLUSIONS

This paper shows that the optimal level of subsidies for bus traffic is substantially lower in a congested bus corridor in the big city than in a small city. The main reason for the different degree of subsidization is the importance of crowding cost relative to waiting and schedule delay cost. However, the optimal fares and subsidies are sensitive to the MCPF. Hence, considering that subsidies require distortive taxes reduces the optimal subsidy level substantially.

The optimal distance between the bus stops is longer in a less dense corridor in the small city. The reason is that the fixed part of the stop time (that is independent from the number of travelers boarding and alighting) is relatively higher in Karlstad because there are fewer passengers boarding at the bus stop.

The optimal bus frequency is, as expected, substantially higher in the congested bus corridor in the big city; this is justified by the bus crowding costs. In the big and congested city, getting the frequency right is as important as getting the fares right for increasing welfare. Due to the big difference in operation costs, crowding and external costs involving an extra bus trip in the big city and the small city, it is not justified to have uniform bus fares throughout the country. In both Stockholm and Karlstad, the policy scenarios are still welfare improving if we take into account income redistribution impacts. Moreover, the bus subsidies are more redistributive in Karlstad than in Stockholm. This is because in Karlstad the low-income citizens use public

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<sup>16</sup> The detailed sensitivity analysis can be found in Appendix H.

transport much more than the rich but in Stockholm there is no large difference in the use of public transport between income groups.

## 6 APPENDICES

All appendices can be found online at: [https://swopec.hhs.se/ctswps/abs/ctswps2018\\_005.htm](https://swopec.hhs.se/ctswps/abs/ctswps2018_005.htm)

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## Appendix A: The average bus line

Table A1: Computing the average bus line of Karlstad.

| Bus line  | Freq (#/h)<br>base case | #stops/km<br>base case | Trip distance (km) | Trip time(min) |
|-----------|-------------------------|------------------------|--------------------|----------------|
| 1         | 6                       | 1.81                   | 16                 | 46             |
| 2         | 6                       | 2.65                   | 20                 | 58             |
| 3         | 6                       | 2.67                   | 12                 | 34             |
| 4         | 3                       | 2.93                   | 15                 | 47             |
| 5         | 3                       | 3.17                   | 6                  | 14             |
| 6         | 3                       | 2.75                   | 12                 | 34             |
| 7         | 3                       | 2.50                   | 8                  | 26             |
| 8         | 3                       | 2.25                   | 16                 | 41             |
| Average   | 4                       | 2.53                   | 13                 | 38             |
|           |                         |                        |                    |                |
| Stockholm | 12                      | 2.58                   | 12                 | 50             |

## Appendix B: Bus cost functions

Table A2: Computing capital and operating cost of buses and bus stops.

| Bus cost functions                           |                              |                              |
|----------------------------------------------|------------------------------|------------------------------|
| Variables                                    | Karlstad                     | Stockholm                    |
| Seats in the bus                             | 45                           | 84                           |
| Bus purchasing cost, €                       | 450 000                      | 500 000                      |
| Depot cost per bus per year, €               | 8 000                        | 8 000                        |
| Life years of bus                            | 12                           | 12                           |
| Discount rate                                | 0.04                         | 0.04                         |
| Days of year                                 | 250                          | 250                          |
| Number of peak trips per bus                 | 5                            | 4                            |
| Yearly capital cost per bus, €               | $47\,948 + 8\,000 = 55\,948$ | $53\,276 + 8\,000 = 61\,276$ |
| Capital cost per peak trip, €                | $55\,948 / 5 / 250 = 45$     | $61\,276 / 4 / 250 = 61$     |
| Capital cost per off-peak trip, €            | 0                            | 0                            |
| Marginal cost for maintenance and fuel, €/km | 1                            | 1                            |
| Breaks for drivers 10% of time               | 1.1                          | 1.1                          |

|                                                   |                                         |                                           |
|---------------------------------------------------|-----------------------------------------|-------------------------------------------|
| Marginal cost for driver, €/km                    | $30 \cdot 38 / 60 \cdot 1.1 / 13 = 1.6$ | $30 \cdot (50 / 60 \cdot 1.1) / 12 = 2.3$ |
| Marginal cost per car trip (maintenance, fuel), € | $(1.6 + 1) \cdot 13 = 34$               | $(2.3 + 1) \cdot 12 = 40$                 |
| Marginal cost per peak trip, €                    | $34 + 45 = 79$                          | $40 + 61 = 101$                           |
| Cost per km, €, peak                              | 6.1                                     | 8.4                                       |
| Cost per km, €, off-peak                          | 2.6                                     | 3.3                                       |
| Bus stops                                         |                                         |                                           |
| Installation, €                                   | 15 000                                  | 15 000                                    |
| Life years of bus stop                            | 10                                      | 10                                        |
| Yearly capital cost per stop, €                   | 1 849                                   | 1 849                                     |
| Yearly operating cost per stop, €                 | 1 000                                   | 1 000                                     |
| Marginal cost bus stop per day, €                 | $(1000 + 1849) / 250 = 11.3$            | $(1000 + 1849) / 250 = 11.3$              |

## Appendix C: Income

The average incomes for the groups can be found in Table A3. In Karlstad the “Retired and others” group is the reference group (and in Stockholm it is the mid income group) and the values of time of the other groups are obtained applying the income elasticity of values of time of 0.7.

Table A3: Income by occupation group, Karlstad and Stockholm. [Source: SCB & CSN]

|                            |            |            |             |
|----------------------------|------------|------------|-------------|
| Karlstad                   | Student    | Retired    | Employed    |
| Average income<br>k€/month | 1.2        | 1.6        | 2.3         |
| Stockholm                  | Low income | Mid income | High income |
| Average income<br>k€/month | 1.12       | 2.8        | 6.1         |

## Appendix D: Calibration and elasticities

We make use of a) observed trip quantities by mode and user group, baseline money costs and time by travel time component; b) the values of time for all travel time components by user group; c) the price and cross-price elasticities listed below and d) speed-flow functions to calibrate this utility function for each user group  $k$ :

$$U^k = m^k + B^{s,k} (q_{c,p}^{s,k}, q_{b,p}^{s,k}, q_{v,p}^{s,k}, q_{c,o}^{s,k}, q_{b,o}^{s,k}, q_{v,o}^{s,k}) + B^{l,k} (q_{c,p}^{l,k}, q_{b,p}^{l,k}, q_{v,p}^{l,k}, q_{c,o}^{l,k}, q_{b,o}^{l,k}, q_{v,o}^{l,k})$$

$$\begin{aligned}
B^{j,k}(q_{c,p}^{j,k}, q_{b,p}^{j,k} q_{v,p}^{j,k}, q_{c,o}^{j,k}, q_{b,o}^{j,k} q_{v,o}^{j,k}) &= a_{c,p}^{j,k} q_{c,p}^{j,k} + a_{b,p}^{j,k} q_{b,p}^{j,k} + a_{v,p}^{j,k} q_{v,p}^{j,k} + a_{c,o}^{j,k} q_{c,o}^{j,k} + a_{b,o}^{j,k} q_{b,o}^{j,k} + \\
&a_{v,o}^{j,k} q_{v,o}^{j,k} - 0.5b_{c,p}^{j,k} (q_{c,p}^{j,k})^2 - 0.5b_{b,p}^{j,k} (q_{b,p}^{j,k})^2 - 0.5b_{v,p}^{j,k} (q_{v,p}^{j,k})^2 - 0.5b_{c,o}^{j,k} (q_{c,o}^{j,k})^2 - 0.5b_{b,o}^{j,k} (q_{b,o}^{j,k})^2 - \\
&0.5b_{v,o}^{j,k} (q_{v,o}^{j,k})^2 - i_{cp,bp}^{j,k} q_{cp}^{j,k} q_{bp}^{j,k} - i_{cp,co}^{j,k} q_{cp}^{j,k} q_{co}^{j,k} - i_{co,bo}^{j,k} q_{co}^{j,k} q_{bo}^{j,k} - i_{bp,bo}^{j,k} q_{bp}^{j,k} q_{bo}^{j,k} - \\
&i_{bp,vp}^{j,k} q_{bp}^{j,k} q_{vp}^{j,k} - i_{bo,vo}^{j,k} q_{bo}^{j,k} q_{vo}^{j,k}.
\end{aligned}$$

The travelers' preferences are represented by a quasi-linear quadratic utility function  $U$ . Utility is the sum of utility derived from non-transport goods (money  $m$ ) and the sub-utility functions  $B^{j,k}$  for all transport goods. The superscripts  $k$  and  $j$  stand for user group and trip length ( $j$ = short [s], long [l]). Short trips are below 4 km and long trips are above 4 km. The travellers of a certain user group and trip length can choose between three modes indicated by the subscripts  $h$  ( $h$  = car [c], bus[b], bicycle [v]), and between the peak ( $p$ ) and the off-peak ( $o$ ) periods. We assume that the demand for short and long trips is independent.  $q$  is the number of trips by the representative individual. The parameters  $i$ ,  $a$  and  $b$  are the parameters in the sub-utility function  $B$ .

In principle, we can have 15 interaction terms ( $i$ ) in total if we take into account all interaction terms across period and across mode. We decide to keep six of these interaction terms in our calibration for two reasons: first, we believe that the interaction terms that we drop are of less importance; second, dropping these terms hugely reduces the computation time because limiting the size of this problem allows us to solve for the parameters ( $a$ ,  $b$ ,  $i$ ) in two steps: first, using Mathematica11, we solve for 12 unknowns ( $b$ ,  $i$ ) out of a system of 12 nonlinear equations (from 12 own-price and cross-price elasticities) in under 20 seconds; then we make use of the 6 user equilibrium conditions for 3 modes and 2 periods to solve for  $a$ . This way, we have perfect control over the elasticities that are of higher importance. All the assumed or implied (generalized price) elasticities result from the calibration are reported below for two of the user groups (other user groups are omitted here due to space limit but are available upon request):

Table A4: Generalized-price elasticities for the short trips, low-income group. The first column are the modes of intervention.

| Karlstad       |           |               |           |               |            |                |
|----------------|-----------|---------------|-----------|---------------|------------|----------------|
|                | Car, peak | Car, off-peak | Bus, peak | Bus, off-peak | Bike, peak | Bike, off-peak |
| Car, peak      | -0.45     | 0.00          | 0.02      | 0.00          | 0.00       | 0.00           |
| Car, off-peak  | 0.00      | -0.63         | 0.00      | 0.02          | 0.00       | 0.00           |
| Bus, peak      | 0.05      | 0.00          | -0.92     | 0.00          | 0.08       | 0.00           |
| Bus, off-peak  | 0.00      | 0.10          | 0.00      | -1.08         | 0.00       | 0.06           |
| Bike, peak     | 0.00      | 0.00          | 0.03      | 0.00          | -0.66      | 0.00           |
| Bike, off-peak | 0.00      | 0.00          | 0.00      | 0.03          | 0.00       | -0.66          |
| Stockholm      |           |               |           |               |            |                |
|                | Car, peak | Car, off-peak | Bus, peak | Bus, off-peak | Bike, peak | Bike, off-peak |
| Car, peak      | -0.61     | 0.01          | 0.01      | 0.00          | 0.00       | 0.00           |
| Car, off-peak  | 0.00      | -0.77         | 0.00      | 0.01          | 0.00       | 0.00           |
| Bus, peak      | 0.09      | 0.00          | -0.70     | 0.00          | 0.23       | 0.00           |
| Bus, off-peak  | 0.00      | 0.16          | 0.01      | -0.68         | 0.00       | 0.23           |
| Bike, peak     | 0.00      | 0.00          | 0.03      | 0.00          | -0.49      | 0.00           |
| Bike, off-peak | 0.00      | -0.01         | 0.00      | 0.03          | 0.00       | -0.49          |

| Karlstad       |           |               |           |               |            |                |
|----------------|-----------|---------------|-----------|---------------|------------|----------------|
|                | Car, peak | Car, off-peak | Bus, peak | Bus, off-peak | Bike, peak | Bike, off-peak |
| Car, peak      | -1.78     | 0.00          | 0.72      | 0.00          | -0.01      | 0.00           |
| Car, off-peak  | 0.00      | -2.45         | 0.00      | 0.93          | 0.00       | -0.01          |
| Bus, peak      | 0.12      | 0.00          | -2.02     | 0.00          | 0.02       | 0.00           |
| Bus, off-peak  | 0.00      | 0.20          | 0.00      | -2.17         | 0.00       | 0.03           |
| Bike, peak     | -0.01     | 0.00          | 0.15      | 0.00          | -3.08      | 0.00           |
| Bike, off-peak | 0.00      | -0.01         | 0.00      | 0.15          | 0.00       | -3.08          |
| Stockholm      |           |               |           |               |            |                |

|                | Car, peak | Car, off-peak | Bus, peak | Bus, off-peak | Bike, peak | Bike, off-peak |
|----------------|-----------|---------------|-----------|---------------|------------|----------------|
| Car, peak      | -3.13     | 0.03          | 0.20      | 0.00          | -0.04      | 0.00           |
| Car, off-peak  | 0.02      | -4.01         | 0.00      | 0.26          | 0.00       | -0.05          |
| Bus, peak      | 0.29      | 0.00          | -2.34     | 0.00          | 0.44       | 0.00           |
| Bus, off-peak  | -0.01     | 0.51          | 0.02      | -2.11         | 0.00       | 0.39           |
| Bike, peak     | -0.03     | 0.00          | 0.23      | 0.00          | -4.32      | 0.00           |
| Bike, off-peak | 0.00      | -0.06         | 0.00      | 0.23          | 0.00       | -4.32          |

## Appendix E: Speed-flow and other parameters

Table A6 shows the  $\beta$  parameters for cars and buses are derived by taking the trip time in the peak and the free-flow travel time, and assuming a linear relationship between the volume of vehicles and the travel time. We might underestimate the travel time delay due to this linear assumption, at least in Stockholm where congestion is more severe. However, the effect of linear assumption should not be exaggerated since we are dealing with relatively small changes around a baseline situation. Moreover, for pricing of bus services and cost recovery it is the congestion cost within the bus and the waiting time externality that have the largest impact, not the congestion function. Still, we might underestimate the fares and the profit of the buses in Stockholm, and also the optimal congestion charges, due to this linear assumption. For the bicycle we assume the free-flow speed to be 12 km/h. Note that the equivalent size of the buses in Karlstad are smaller, because they have fewer seats than the Stockholm buses (45 vs 84 seats).

Table A6: Speed-flow and other parameters.

| Parameter                                                                     | Notation   | Karlstad | Stockholm |
|-------------------------------------------------------------------------------|------------|----------|-----------|
| Free-flow time in min/trip, car                                               | $\alpha_c$ | 16       | 22        |
| Number of car lanes                                                           | $cap$      | 1.5      | 1.5       |
| Free-flow time in min/trip, bus                                               | $\alpha_b$ | 26       | 30        |
| Free-flow time in min/trip, bicycle                                           | $\alpha_v$ | 65       | 60        |
| Additional time needed for one more vehicle, car (min per vehicle equivalent) | $\beta_c$  | 0.01     | 0.0385    |

|                                                                                    |               |       |       |
|------------------------------------------------------------------------------------|---------------|-------|-------|
| Additional time needed for one more vehicle, bus (min per vehicle equivalent)      | $\beta_b$     | 0.01  | 0.03  |
| Additional time needed for one more vehicle, bicycle (min per vehicle)             | $\beta_v$     | 0.001 | 0.001 |
| Car trips slowing down at bus stops, if interacting with bus (min per vehicle)     | $\gamma_c$    | 0.02  | 0.02  |
| Bus delay at bus stops, if interacting with a bicycle (min per vehicle)            | $\gamma_b$    | 0     | 0.02  |
| Delay for bicycles at bus stops, if interacting with car and bus (min per vehicle) | $\gamma_v$    | 0     | 0.02  |
| Minimum stop time at a bus stop (sec)                                              | $t_s$         | 10    | 10    |
| Boarding/alighting time (sec/passenger)                                            | $\zeta$       | 1.8   | 1.8   |
| Time cars/cyclists passing a bus stop (sec)                                        | $TS_c = TS_v$ | 2     | 2     |
| Walking speed (km/h)                                                               | $v_w$         | 6     | 6     |
| Discomfort coefficient for bus trip                                                | $dc_b$        | 1.1   | 1.1   |
| Equivalent size of bus to cars                                                     | $\sigma(V_b)$ | 2.5   | 5     |
| On-board ratio (peak and off-peak)                                                 | $r_{on}$      | 0.65  | 0.22  |

## Appendix F: Fares (MCPF=1.2)

Table A7: Fares and tolls in the optimal pricing scenarios, MCPF = 1.2. (p: peak, o: off-peak)

| Policy scenario                 | Short bus trips | Long bus trips | Short car trips | Long car trips | Short bicycle trips | Long bicycle trips |
|---------------------------------|-----------------|----------------|-----------------|----------------|---------------------|--------------------|
| <b>Karlstad</b>                 |                 |                |                 |                |                     |                    |
| Baseline (p)                    | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| (o)                             | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| 1. All prices + freq +stops (p) | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| (o)                             | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| 2. All prices only (p)          | 1.4             | 1.6            | 1.0             | 1.6            | 0.7                 | 0.7                |
| (o)                             | 1.3             | 1.3            | 0.6             | 0.9            | 0.7                 | 0.7                |
| 3. Frequency +stops (p)         | 1.4             | 1.5            | 1.0             | 1.6            | 0.7                 | 0.7                |
| (o)                             | 1.3             | 1.3            | 0.6             | 0.9            | 0.7                 | 0.7                |
| 4. =1+zero tolls (p)            | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| (o)                             | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |

|                                      |     |     |     |     |     |     |
|--------------------------------------|-----|-----|-----|-----|-----|-----|
| 5. =1 + bus lane + zero<br>tolls (p) | 1.1 | 1.2 | 0.0 | 0.0 | 0.0 | 0.0 |
| (o)                                  | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| <b>Stockholm</b>                     |     |     |     |     |     |     |
| Baseline (p)                         | 2.0 | 2.0 | 1.8 | 1.8 | 0.0 | 0.0 |
| (o)                                  | 2.0 | 2.0 | 1.0 | 1.0 | 0.0 | 0.0 |
| 1. All prices + freq<br>+stops (p)   | 1.7 | 1.9 | 2.3 | 4.8 | 0.7 | 0.8 |
| (o)                                  | 1.5 | 1.6 | 1.2 | 2.5 | 0.6 | 0.7 |
| 2. All prices only (p)               | 1.9 | 2.1 | 2.3 | 4.8 | 0.7 | 0.8 |
| (o)                                  | 1.5 | 1.6 | 1.2 | 2.5 | 0.6 | 0.7 |
| 3. Frequency +stops (p)              | 2.0 | 2.0 | 1.8 | 1.8 | 0.0 | 0.0 |
| (o)                                  | 2.0 | 2.0 | 1.0 | 1.0 | 0.0 | 0.0 |
| 4. =1+zero tolls (p)                 | 1.5 | 1.7 | 0.0 | 0.0 | 0.0 | 0.0 |
| (o)                                  | 1.3 | 1.4 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5. =1+no bus lane+zero<br>tolls (p)  | 1.2 | 1.2 | 0.0 | 0.0 | 0.0 | 0.0 |
| (o)                                  | 1.2 | 1.2 | 0.0 | 0.0 | 0.0 | 0.0 |

## Appendix G: Weighted welfare function

Now we move from analysing the redistributive effects of subsidies only, to a more general analysis of the reforms designed by optimizing fares and frequencies. The simple method used in subsection 4.3 cannot be applied to reforms with more dimensions than subsidies, and we therefore apply a weighted welfare function. This has two more advantages. First, the latter method measures the effect on welfare and not euros of subsidies and who pays for it (even if the marginal change in fares impacts the travel behaviour which disregarded above). Second, with more income groups the effects of a change in prices can very well be non-monotonic with income. When the high- and low-income group gain but the middle-income group loses, one needs to weight the gains and losses per income group. However, since we use a different metric the redistributive effects cannot be quantified in monetary terms as easily as in the simpler analysis.

We compare how the distributionally-weighted welfare changes with each reform with respect to the reference equilibrium. If this is positive, the reform is favourable to the equity weighted welfare of

society. We consider a reference equilibrium indexed  $^{\circ}$  and an alternative equilibrium indexed  $^*$ . The equity weighted welfare function in the reference equilibrium and the change in weighted welfare are, respectively,

$$\Omega^{\circ}(\theta) = \sum_k n^k \theta^k (m^k)^{\circ} + \sum_k n^k \theta^k \cdot \sum_j [B^{j,k}(\dots, (q_h^{j,k})^{\circ}, \dots)] - \sum_k n^k \theta^k \cdot (S^k)^{\circ} + \sum_k n^k \theta^k \cdot \sum_j (E_h^{j,k})^{\circ} \quad (1)$$

$$\Omega^*(\theta) - \Omega^{\circ}(\theta) = \sum_k n^k \theta^k [(m^k)^* - (m^k)^{\circ}] + \sum_k n^k \theta^k \cdot \sum_j [B^{j,k}(\dots, (q_h^{j,k})^*, \dots) - B^{j,k}(\dots, (q_h^{j,k})^{\circ}, \dots)] - \sum_k n^k \theta^k \cdot [(S^k)^* - (S^k)^{\circ}] + \sum_k n^k \theta^k \cdot \sum_j [(E_h^{j,k})^* - (E_h^{j,k})^{\circ}], \quad (2)$$

where  $\theta^k$  represents the distributional weight of income group  $k$ ,  $S^k$  represents the contribution of income group  $k$  to the financing of the government deficit associated to the transport sector, and  $E_h^{j,k}$  stands for the external costs, other than congestion, that are affecting income group  $k$ . We assume that the incidence of the external costs for each income group is distributed uniformly over the population.

To implement this weighted welfare function, we again assume that all toll revenues and fare revenues minus bus operation costs are distributed equally to each citizen (head tax or subsidy). Also, we need to assign weights to different income groups.

The welfare weights  $\theta^k$  are based on value judgements but economists try to apply a consistent set of weights: the weights can be defined using an analytical expression that incorporates inequality aversion ( $\varepsilon$ ): when there is no inequality aversion, the weights are all equal (we have our original welfare function again). When there is inequality aversion, it is convenient to use a welfare function that is a sum of weighted individual utility functions. The weights are chosen such that the marginal contribution to the sum of the individuals' welfare of one euro is decreasing with the income of the individual who receives it. We use the following iso-elastic function where  $U$  is the utility function we used previously and that has the dimension income equivalent

$$\Omega^k = \frac{(U^k)^{1-\varepsilon}}{1-\varepsilon} \quad (3)$$

and the corresponding definition of welfare weights is

$$\frac{\theta^k}{\theta^r} = \left( \frac{U^r}{U^k} \right)^\varepsilon \quad (4)$$

where the index  $r$  represents the reference income group and  $\theta^r$  is normalized to 1. The inequality aversion ( $\varepsilon$ ) can be 0, then there is no aversion to inequality and then all weights are equal. A stronger value is  $\varepsilon = 1$ , then one uses the inverse income inequality. A value that is often used is 0.5: this implies that one is indifferent between giving 1 euro to one person and giving 0.5 euro to somebody that is 4 times richer. Moreover, the sum of weights of the three income groups is normalized to 1.

We now analyse the welfare effects arising from the reforms derived in section 4.1 while changing the relative weights assigned to the income groups (i.e. changing  $\varepsilon$  from 0 to 0.5 and 1). We do this by comparing the weighted welfare in the situation after the implementation of the reforms to the weighted welfare in the baseline (i.e. the scenario values (fares, frequency etc.) are still the ones obtained with the simple unweighted welfare optimisation). Hence we analyse the same reforms as before from different welfare perspectives. However, since a value of Euro is a weighted sum of Euros, a different metric is used, and results can therefore not be compared across columns. Still, the ranking of policies may change when the relative weights given to the different income groups vary.

The ranking of the three reforms stays the same in Table A8. The total welfare change is still positive for all scenarios and for the two welfare weightings we use. We can conclude that these policies are welfare improving when higher weights are assigned to lower-income groups, so these policies are justifiable even when the government has distributional objectives.

Table A8: Weighted welfare in different scenarios.

| Scenarios | $\varepsilon = 0$  | $\varepsilon = 0.5$ | $\varepsilon = 1$  |
|-----------|--------------------|---------------------|--------------------|
|           | Change in weighted | Change in weighted  | Change in weighted |

|                      | welfare (per day) | welfare (per day) | welfare (per day) |
|----------------------|-------------------|-------------------|-------------------|
| <b>Karlstad</b>      |                   |                   |                   |
| 1. All optimized     | 679               | 596               | 499               |
| 2. All prices        | 526               | 426               | 310               |
| 3. Frequency + stops | 170               | 178               | 187               |
| <b>Stockholm</b>     |                   |                   |                   |
| 1. All optimized     | 10056             | 9691              | 9311              |
| 2. All prices        | 3608              | 3588              | 3567              |
| 3. Frequency + stops | 4973              | 4756              | 4530              |

## Appendix H: Sensitivity analysis

Table A9 and Table A10 show the sensitivity analysis with cross-elasticities and discomfort coefficient. In general the results are reassuringly robust, both in terms of optimal bus fares and the percentage of subsidies. As expected, increasing the discomfort coefficient to 1.2 increases the bus frequencies, but the subsidy rates are not too different, because the increase in supply cost is compensated by the increase in optimal bus fares.

Table A9: Sensitivity analysis of the scenario 1 (all optimized, MCPF=1). (p: peak, o: off-peak)

| Scenario                    | Number of stops /km | Frequency /hour | Supply cost €/day | Subsidy % | Total number of bus trips / day |
|-----------------------------|---------------------|-----------------|-------------------|-----------|---------------------------------|
| <b>Karlstad</b>             |                     |                 |                   |           |                                 |
| Baseline (p)                | 2.54                | 4.00            | 7704              | 41        | 1442                            |
| (o)                         |                     | 2.00            |                   |           | 845                             |
| Main analysis (p)           | 1.89                | 2.93            | 6621              | 83        | 1767                            |
| (o)                         |                     | 2.26            |                   |           | 1223                            |
| 2*diversion car and bus (p) | 1.88                | 3.00            | 6749              | 82        | 1824                            |
| (o)                         |                     | 2.30            |                   |           | 1250                            |
| Discomfort=1.2 (p)          | 1.85                | 3.05            | 6810              | 77        | 1703                            |
| (o)                         |                     | 2.30            |                   |           | 1211                            |
| <b>Stockholm</b>            |                     |                 |                   |           |                                 |
| Baseline (p)                | 2.58                | 12.00           | 35164             | -270      | 32500                           |
| (o)                         |                     | 11.00           |                   |           | 32500                           |

|                             |      |       |       |   |       |
|-----------------------------|------|-------|-------|---|-------|
| Main analysis (p)           | 2.21 | 21.68 | 59604 | 6 | 43367 |
| (o)                         |      | 17.23 |       |   | 43667 |
| 2*diversion car and bus (p) | 2.20 | 21.70 | 59643 | 9 | 43380 |
| (o)                         |      | 17.23 |       |   | 43625 |
| Discomfort=1.2 (p)          | 2.18 | 23.43 | 63642 | 3 | 43511 |
| (o)                         |      | 18.09 |       |   | 43571 |

Table A10: Sensitivity analysis (optimal prices). (p: peak, o: off-peak)

| Scenarios                   | Short bus trips | Long bus trips | Short car trips | Long car trips | Short bicycle trips | Long bicycle trips |
|-----------------------------|-----------------|----------------|-----------------|----------------|---------------------|--------------------|
| <b>Karlstad</b>             |                 |                |                 |                |                     |                    |
| Baseline (p)                | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| (o)                         | 2.0             | 2.0            | 0.0             | 0.0            | 0.0                 | 0.0                |
| Main analysis (p)           | 0.3             | 0.6            | 0.4             | 1.2            | 0.0                 | 0.1                |
| (o)                         | 0.1             | 0.2            | 0.2             | 0.7            | 0.0                 | 0.0                |
| 2*diversion car and bus (p) | 0.4             | 0.6            | 0.4             | 1.2            | 0.0                 | 0.1                |
| (o)                         | 0.1             | 0.2            | 0.2             | 0.6            | 0.0                 | 0.0                |
| Discomfort=1.2 (p)          | 0.4             | 0.9            | 0.4             | 1.2            | 0.0                 | 0.1                |
| (o)                         | 0.2             | 0.3            | 0.2             | 0.7            | 0.0                 | 0.0                |
| <b>Stockholm</b>            |                 |                |                 |                |                     |                    |
| Baseline (p)                | 2.0             | 2.0            | 1.8             | 1.8            | 0.0                 | 0.0                |
| (o)                         | 2.0             | 2.0            | 1.0             | 1.0            | 0.0                 | 0.0                |
| Main analysis (p)           | 0.6             | 0.8            | 1.0             | 4.0            | 0.1                 | 0.3                |
| (o)                         | 0.4             | 0.5            | 0.5             | 2.1            | 0.0                 | 0.1                |
| 2*diversion car and bus (p) | 0.6             | 0.8            | 1.0             | 4.0            | 0.1                 | 0.3                |
| (o)                         | 0.4             | 0.5            | 0.5             | 2.0            | 0.0                 | 0.1                |
| Discomfort=1.2 (p)          | 0.7             | 1.0            | 1.0             | 4.0            | 0.1                 | 0.3                |
| (o)                         | 0.5             | 0.6            | 0.5             | 2.1            | 0.0                 | 0.1                |