Welfare Effects of Open-Access Competition on Railway Markets
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Abstract
In recent years, several countries have deregulated passenger railway markets to allow open access. The aim is for competition to lower fares and increase quality of service, thereby increasing demand, economic efficiency and overall social welfare. In this paper, we use a stylised simulation model to study how open access competition affects fares, demand, supply, consumer surplus and operator profits compared to a profit-maximising monopoly and to a welfare-maximising benchmark situation. We conclude that aggregate social welfare increases substantially when going from profit-maximising monopoly to duopoly competition, as consumers make large gains while operators’ profits fall. According to simulations, there generally exists a stable competitive Nash equilibrium with two or more profitable operators. Although operators are identical in the model setup, the Nash equilibrium outcome is asymmetric: one operator has more departures and higher average fares than the other. If operators are allowed to collude, however, for example by trading or selling departure slots, the equilibrium situation tends to revert to monopoly: it will be profitable for one operator to buy the other’s departure slots to gain monopoly power. The regulatory framework must therefore prevent collusion and facilitate market entry. Even the potential for competitive entry tends to increase social welfare, as the monopolist has incentives to increase supply as an entry deterrence strategy.

Keywords: Open access, rail, reform, capacity allocation, passenger

JEL codes: D43, R41, R48

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1. Introduction

Open access competition has been introduced in several European passenger railway markets in recent years. The idea is that several operators compete for passengers on one line, with the aim to increase welfare through efficiency gains, price pressure and improved services, leading to increased demand. Considering the trend towards open access competition, there is a clear need to assess the magnitude of these improvements, and to offer policy advice regarding how to maximise welfare under the regime. To do so, the most prominent features of open access railway markets must be taken into account. Among these are, first, that a regulator (the infrastructure manager) must decide a timetable based on several competing operators’ potentially conflicting capacity requests; and second, that there is a natural product differentiation in that each train service has its own, specific departure time, so it is usually impossible for two competing operators to supply the exact same service. As we shall see, these features have important implications for the market outcomes.

We use a stylised simulation model to explore the dynamics created by the market’s most prominent features. Using simulation allows us to study a richer model than can be analysed with purely analytic methods, while the simplicity of the model supports comprehension of the causes for the (sometimes surprising) results of the simulation. The results are tested to hold for a range of parameter values (see Appendix C.). The welfare gains of replacing a profit-maximising monopoly with oligopolistic competition is assessed and compared to a welfare-maximising benchmark. In the model, two (or more) competing operators announce their desired departure frequencies, and the regulator (the infrastructure manager) then arranges the departures (we show that arranging them in alternating order maximises welfare). Conditional on this departure pattern, operators set specific fares for each departure to maximise their total profits (mutually taking each other’s fares into account). Frequencies are determined to reach a Nash equilibrium (we also study the Stackelberg equilibrium). The model hence reflects several specific features for the railway market: the infrastructure manager has a certain power to regulate the eventual timetable; competition occurs not only between operators but between departures that are close in time; and specific fares can be set for each individual departure by the operator.

Results indicate that duopoly competition substantially increases aggregate social welfare, compared to a profit-maximising monopoly. The consumer surplus increases while profits decrease, due to lower average fares and higher service frequency, and the net social effect of this is positive and substantial. However, if profits in the competitive situation are transferred abroad, for example if a national monopoly operator is replaced by foreign-owned operators, replacing the monopoly with competition in fact tends to reduce overall domestic welfare. Assuming free entry, a stable Nash equilibrium is reached with two (or more) operators making positive profits. Despite operators being identical in the model, the equilibrium tends to consist of one dominating operator supplying more departures and higher average fares, and one small operator with fewer departures and lower average fares. The combined profit of the competing operators is substantially lower than the monopoly profit, meaning that there is an incentive to merge operations into a single unit, or for one operator to buy the other’s departure slots. Constructing a regulatory framework that prevents the market reverting into a monopoly is not trivial.

A preliminary version of some of the results in this paper were presented in (Broman & Eliasson, 2017). The present paper contains a refinement of that model, including an updated expression for consumer surplus and a more realistic cost-structure for operators; the latter affects the dynamics of the model. A new benchmark-scenario is included: a monopolist that operates under an entry-deterrence strategy. We also analyse how the infrastructure manager should construct a timetable out of operators’ requests, in order to maximise welfare. Additional policy implications are drawn from the refined model and new benchmark case.

2. Background

Over the last few decades, many railway markets have become increasingly deregulated, especially in Europe. Most European countries nowadays have separated service provision from infrastructure management, meaning that one or several operators run the trains, determine fares and (to a large extent) determine service frequencies, while an infrastructure manager (usually a government agency) is responsible for track maintenance and investments, and also for solving possible conflicts between competing operators’ service requests. Some operators are companies or agencies controlled by the government, while some operators are privately owned companies. Sweden was among the first companies to deregulate its railway system, separating service operations from infrastructure management in 1989. The UK has also come far in this respect, with public tenders for all lines. In one way or another, the deregulation trend has spread throughout Europe and beyond.

A few countries are now taking this one step further, through introducing competition not just for the tracks but on the tracks – so-called open access competition. Since 2001, the Swedish freight market is completely...
deregulated, and since 2010, also the passenger market; open access competition is the market regime for all profitable parts of the network (services on unprofitable lines are procured through public tendering) (Alexandersson & Hultén, 2009). Other countries that are experimenting with open access competition include Austria, the Czech Republic, Italy and the UK (Beria, Redondi, & Malighetti, 2014). The result has in most cases been a duopoly situation, sometimes complemented by smaller niche actors. Our analysis in this paper focuses on passenger markets.

More is known about the effects of open access on other modes than rail. When the British bus market was deregulated in the 1980s, new entrants challenged the incumbent only on a small share of submarkets. Where they did, this led to a short period of fierce competition in price as well as frequency. Profitability for both competitors rapidly sank well into the negative and within a year or so one of them closed shop. At that point fares increased again and departure frequencies decreased, although prices remained lower and frequencies higher compared to before deregulation – possibly as a deterrence strategy against competition. (Evans, 1990). Simulations of the bus market reach similar conclusions (see e.g. (Evans, 1987)).

Railway markets differ from bus markets in some important respects, however, which affects outcomes. Whereas prices are set freely, railway timetables are partly determined by the infrastructure manager, which is usually a government agency. Moreover, timetables can usually only be changed at certain points in time, common for all operators. Therefore, the market dynamics can be described by a three-step process: First, operators apply for capacity; then, departure slots are allocated; finally, prices are set in competition, with timetables fixed for a certain time period.

Another difference is that individual departures have different prices, which is unusual in bus markets. In fact, individual railway tickets are often sold at different prices, due to the prevalence of yield management price-setting software. Also partly because of such software, prices can be adapted quickly when needed, possibly removing the reason for a monopolist to lower prices as an entry deterrence strategy. In contrast, deterrence strategies in frequency seem more plausible, as building the necessary capacity is a slower process.

3. The model

The simulation model used in this paper is constructed to capture some essential features of open access competition in passenger railway markets. We consider only one origin-destination pair, and all train services have the same running time; they only differ in terms of departure time and fare. Passengers choose which train service to travel with (if any) by minimising their generalised travel cost, which is the sum of the fare and the schedule delay (the difference between their preferred departure time and the respective service’s departure time). Demand is a function of generalised cost.

Operators have identical production cost functions, and strive to maximise profits by choosing departure frequencies and fares. These are determined in a two-stage process: First, operators announce their desired frequencies. The regulator (the infrastructure manager) determines the timetable based on these frequencies, ordering operators’ departures to maximise social welfare. Second, operators decide the fares of each departure to maximise their respective total profit under the given timetable, taking the fares of competing operators into account. Fares are hence determined by a Nash equilibrium. Operators are assumed to understand what the outcome of the second step will be from the beginning, and choose frequencies in the first step accordingly. Nash equilibrium is therefore reached in both the first step (frequencies) and the second step (fares).

Through this design, the model captures some key characteristics that distinguish railway markets from that of e.g. buses. The infrastructure manager is by necessity involved in timetabling, and strives to maximise welfare (which is not necessarily aligned with operators’ interests). Fares and timetables are determined in a two-step process. The timetable is based on operators’ requests, but the details are determined by the infrastructure manager, and lasts for a certain period of time (such as a year). Fares are set conditional on the timetable, can be changed relatively easy, and are set freely by operators.

3.1. Calculating demand

Every potential passenger has a unique preferred departure time (PDT) t; in all other respects passengers are identical. The PDTs have a distribution φ(t) over the day such that \( \int \phi(t) dt \) equals the daily potential demand. There are \( N \) trains departing at times \( T_1, T_2, \ldots, T_N \) with different fares \( p_1, p_2, \ldots, p_N \). Trains are assumed to have unlimited capacity, and the marginal passenger cost for operators is zero. Passengers choose the departure that minimises their generalised travel cost, which is the sum of the fare and the monetary value of their schedule delay.

\[ 1 \] This may be substituted with preferred arrival time with analogous results.
delay, i.e. the difference between the departure time \( T_n \) and the PDT \( t \). We assume that the monetary valuation of the schedule delay is constant and symmetric, so the minimal generalised travel cost for a passenger with PDT \( t \) is

\[
c(t) = \min\limits_n (p_n + \alpha |T_n - t|)
\] (1)

Demand depends linearly on travellers’ generalised cost. The number of passengers with PDT \( t \) that choose to travel is

\[
D(t) = \max\{\varphi(t) - \beta c(t), 0\},
\] (2)

where \( \beta > 0 \).

To calculate the number of passengers choosing to travel with departure \( n \), let \( \tau_n \) be the PDT of a passenger who is indifferent between departures \( T_n \) and \( T_{n+1} \). This means that passengers with PDTs in the interval \([\tau_{n-1}, \tau_n]\) will travel with train service \( n \). Let \( \tau_0 \) be the start of the day and \( \tau_N \) the end of the day. Straightforward calculations give

\[
\tau_n = \frac{p_{n+1} - p_n}{2\alpha} + \frac{T_{n+1} + T_n}{2}, n = 1, ..., N - 1
\] (3)

The number of passengers who travel with train service \( n \) is then

\[
D_n = \int_{\tau_{n-1}}^{\tau_n} D(s)ds
\] (4)

3.2. Consumer and producer surplus

Using equation (2), the consumer surplus conditional on PDT \( t \) becomes (the derivations can be found in Appendix A.)

\[
CS(t) = \frac{1}{2\beta} D(t)^2 = \frac{(\varphi - \beta p_n)^2}{2\beta} + \frac{\alpha^2 \beta}{2} (T_n - t)^2 + \alpha(\beta p_n - \varphi)|T_n - t|
\] (5)

Assuming that \( \varphi(t) \) is constant throughout the day (which we will do in the simulations) the consumer surplus becomes

\[
CS = \sum_n \int_{\tau_{n-1}}^{\tau_n} CS(t)dt = \sum_n \frac{(\varphi - \beta p_n)^2}{2\beta} (\tau_{n+1} - \tau_n) + \frac{\alpha^2 \beta}{6} ((T_n - \tau_n)^3 + (\tau_{n+1} - T_n)^3) + \frac{\alpha(\beta p_n - \varphi)}{2} ((T_n - \tau_n)^2 + (\tau_{n+1} - T_n)^2)
\] (6)

The profit for each operator is the sum of net profits for all its departures during a day:

\[
\Pi_k = \sum_{n \in S_k} (p_n D_n(p) - K) - \kappa
\] (7)

where \( S_k \) is the set of train services run by operator \( k \), \( p_n \) is the fare for departure \( n \), \( K \) is the operations cost per departure and \( \kappa \) is the operator’s fixed cost. Each operator chooses its fares to maximise \( \Pi \), conditional on the fares of the other operator, so that fares are decided by the Nash equilibrium.

The total social welfare is the sum of the consumer surplus and the producer surplus. The model hence does not include any external benefits of traveling. The conventional assumption that producers’ surplus is included in
the overall social welfare is not innocuous, since train operators may well be foreign owned and a decision maker in a country may well consider producer surplus accruing to foreign owners as “lost” from a domestic perspective (compared to, say, a government-owned operator, where profits accrue to domestic tax payers). In the analysis below, we will also discuss how this perspective may change some of the conclusions.

3.3. Choosing frequencies and fares

The game is designed to reflect an important aspect of railway markets: that frequencies are set rarely, while fares are set continuously. It is played as follows.

There is a limited number of operators. First, they decide their respective frequency, i.e. how many departures per day they will run. An equilibrium point is reached where no operator wishes to change its chosen frequency. Then the infrastructure agency decides the exact timetable so that there is equal space between any two consecutive departures, and departures of different operators are intermingled in such a way that passengers have as many options as possible throughout the day. (We will demonstrate that this is the welfare-maximising way to arrange the departures, given a uniform PDT distribution.)

Secondly, operators set fares for each departure. The sub-game ends in Nash equilibrium, where no operator wishes to change the fare on any of its departures given the timetable and the fares of its competitor(s). The entire process is reiterated until both the frequency and the price game are in equilibrium.

Note that fares affect demand in two ways: decreasing a fare on a specific departure will both increase the total number of passengers and attract passengers from adjacent departures. We assume that operators take into consideration the ownership of adjacent departures when setting fares. This means that the general price level depends not only on the frequency of departures but also on the how departures are ordered.

3.4. Parameters

The purpose of a stylised simulation as the one used here is obviously not to predict quantitative outcomes but to gain insights. Still, the model parameters are calibrated to give outcomes resembling a real case, which means that we have sufficient confidence in the relative magnitudes of the results. Moreover, the conclusions reported in the paper have been tested for robustness through parameter sensitivity analysis. Appendix C presents some key results from the main sensitivity analyses.

The model parameters have been calibrated so the outcome resembles the Stockholm-Gothenburg railway line: ca 5,000 daily trips, between 10 and 30 departures per direction, and fares mainly in the interval of 200-1,000 SEK. In the base case, the parameters are scheduling cost $\alpha = 500$ SEK/hour, demand/price parameter $\beta = -10$, potential demand $\int \varphi(t) \, dt = 15,000$ passengers/day, fixed cost per departure $K = 40,000$ SEK and fixed daily cost per operator $\kappa = 500,000$ SEK. For simplicity, the PDT distribution $\varphi(t)$ is taken to be a uniform distribution; this is simply to make model outcomes easier to interpret As will be shown below, this means that the regulator (which strives to maximise social welfare) will place the train departures at regular distances in time.

4. Analysis

This section presents simulation results. Section 4.1 presents results for monopoly situations, which serve as benchmarks to be compared with situations with competing operators. Section 4.2 presents results for two competing operators in Nash equilibrium. Section 4.3 shows that the infrastructure regulator, which decides the order and headway of departures given operators’ choice of frequencies, should mix departures of competing operators as much as possible, and spread departures evenly to maximise welfare. Throughout the analyses of competitive situations, we will assume that this is what the regulator does. Section 4.4 analyses situations with more than two operators: first, a situation with many potential operators and free entry, and second, a situation with two operators using an entry deterrence strategy to keep a potential third operator from wanting to enter the market.

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2 The scheduling cost $\alpha$ is based on the value of headway in (ASEK6), assuming a 25% ratio of business trips.

3 The cost per departure is calculated as $K = \gamma_1 \cdot \text{travel time} + \gamma_2 \cdot \text{travel distance}$ where the parameters $\gamma_1$ and $\gamma_2$ are taken from (ASEK6)
4.1. Monopoly

Consider three monopoly situations: welfare maximisation with a cost recovery constraint (i.e. no operator subsidy), profit maximisation, and entry deterrence. The first, welfare maximisation under cost recovery, will serve as a benchmark for subsequent comparisons; it can be interpreted as a publicly controlled company with perfect information, only serving the interests of society (disregarding any potential problems with public monopolies, e.g. of internal efficiency). The second, profit maximisation, is simply a standard monopoly situation: the monopolist chooses fares and service frequency to maximise its total profit. The third situation, entry deterrence, builds on the assumption that fares can be adapted quickly while frequencies cannot: an incumbent monopolist chooses a frequency high enough to deter a potential competitor from entering the market, and given this frequency, the monopolist is then free to choose profit-maximising fares. Both the incumbent and a potential competitor anticipate that if there are two competing operators, fares will instead be Nash equilibrium prices. It is often impossible for the incumbent to prevent competitive entry with certainty; instead, it can take measures to make it less profitable and more expensive. Such a strategy has no ‘optimum’ in the normal sense, why we have instead included one possible strategy that makes competitive entry less desirable. Table 1 shows results from the simulation of the first two situations, welfare maximisation and profit maximisation, and a possible entry deterrence strategy.

Welfare maximisation (without subsidies) leads to 22 departures per day, and an average fare of SEK104, which is just enough to make total profit non-negative. Compared to the profit-maximising monopoly, there are six more departures, and the average fare is 85% lower. Demand is almost doubled and consumer surplus is almost four times larger. Total social welfare is just over 40% higher, since the gain in consumer surplus is partly offset by a decrease in producer surplus. Obviously, these particular figures will depend on the specific parameters of the model, in particular the assumed demand elasticity and the operator’s cost structure. But since the model parameters have been chosen from realistic situations, the figure is likely to be in a realistic order of magnitude. (Parameter sensitivity analyses are presented in Appendix C.)

Table 1. Monopoly situations.

<table>
<thead>
<tr>
<th></th>
<th>Welfare max., no subsidy</th>
<th>Profit max.</th>
<th>Entry deterrence (example)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of departures (per day)</td>
<td>22</td>
<td>16</td>
<td>25</td>
</tr>
<tr>
<td>Average fare (SEK)</td>
<td>104</td>
<td>705</td>
<td>720</td>
</tr>
<tr>
<td>No. of passengers</td>
<td>13,300</td>
<td>7,100</td>
<td>7,200</td>
</tr>
<tr>
<td>Consumer surplus (SEK)</td>
<td>8,840,000</td>
<td>2,430,000</td>
<td>2,570,000</td>
</tr>
<tr>
<td>Producer surplus (SEK)</td>
<td>0</td>
<td>3,800,000</td>
<td>3,680,000</td>
</tr>
<tr>
<td>Total welfare (SEK)</td>
<td>8,840,000</td>
<td>6,230,000</td>
<td>6,260,000</td>
</tr>
</tbody>
</table>

In the entry deterrence scenario, the incumbent increases the service frequency to avert competitors from entering the market by making it difficult or unprofitable. If the number of incumbent departures increases, an entrant would also need a larger number of departures to gain the market share needed to pay for fixed costs. As a larger number of departures requires a larger investment in rolling stock, this makes market entry riskier. If the incumbent runs 16 departures, which is the profit-maximising frequency, a prospective entrant would need to run at least 3 departures to be profitable. If the incumbent increases its frequency to 25 departures, a prospective entrant would need to run at least four departures to be profitable, and its highest attainable profit would be reduced by 58% compared to the scenario where the incumbent runs 16 departures. The incumbent, on the other hand, does not lose much by increasing its frequency from 16 to 25 departures: its total profits only decrease by 3%. This is hence a cheap and reasonably effective deterrence strategy for the incumbent. Social welfare increases somewhat by this increase in frequency, but only slightly (less than 0.5%), since the incumbent can still charge monopoly fares as long as it is the only operator. As a corollary, we can note that if a regulator wants to reduce the negative effects of a monopoly, it is not a very effective strategy to force the monopolist to increase its frequency, as long as the operator is free so set monopoly fares.

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\footnote{Strict welfare maximisation would lead to negative profits, since the optimal fare is equal to the marginal passenger cost, and hence revenues will not cover fixed operations costs. While subsidised train services are common for intra-regional public transport (commuter trains), it is much less common for long-distance train markets, which is the focus of this paper, so we will not consider this situation.}
4.2. Frequency equilibrium under duopoly

In the duopoly situation, we assume that the two operators announce how many departures they want to run, and the infrastructure regulator then translates this into a timetable, i.e. it decides the headway and ordering of departures. The infrastructure regulator strives to maximise overall social welfare when doing this. In section 4.3, we will show this means that the regulator will mix departures run by different operators as much as possible, and spread departures evenly across the day (since we have assumed a uniform PDT distribution).

Given this timetable, operators choose profit-maximising fares until they reach a Nash equilibrium. The operators know the outcome of this subgame when they decide their number of departures, and their choices on number of departures also lead to a Nash equilibrium.

The frequency Nash equilibrium is not unique, however, as there are two symmetric equilibria: if operator 1 running X trains and operator 2 running Y trains is a Nash equilibrium, then operator 1 running Y trains and operator 2 running X trains is also a Nash equilibrium, since operators are indistinguishable in our model setup. (As we shall see, X is generally not equal to Y in Nash equilibrium.) Results from the simulation model are shown in Table 2, compared with the two benchmark monopoly situations from section 4.1.

Figure 1 shows fares, demand and profits in Nash equilibrium. One operator has 18 departures and the other has 7. Note that the total number of departures is higher in the duopoly case than in welfare-maximising monopoly (25 departures rather than 22) in the base case. The intuition for this is that although a higher frequency generates value for travellers through reduced scheduling costs, it also requires fares to rise in order to pay for the extra trains, and the fare increase is larger than the scheduling cost reduction. The welfare maximum thus has fewer departures.

On most departures, the entrant offers lower fares and consequently attracts more passengers per departure; it makes a higher profit per departure than the incumbent. The departures controlled by the incumbent have higher fares and lower profits on average. In particular, the incumbent’s departures which have other incumbent departures on either side have the highest fares, lowest demand and yield lowest profit. In fact, the profit of these departures would increase if fares were lowered – but most of the attracted demand would come from other departures controlled by the incumbent, so the incumbent’s total profit would decrease. This shows how the overall outcome is affected by a market setup where operators are able to control the fare of each individual departure, and maximises the aggregate profit from all their departures, rather than the profit of each departure seen in isolation. This is why it matters greatly how the departures of different operators are ordered, as we shall see in section 4.3.

The operators’ ability to control fares individually for each departure, and to take into account whether a departure competes with other departures controlled by the same operator or those of a competitor, is also the reason why the Nash equilibrium is asymmetric in frequencies, with one “large” operator running many departures and one “small” operator running fewer. Remember that the operators in the model setup are identical, and that there are no economies of scale in production costs in the model (apart from a fixed cost to enter the market at all). Given this, one might expect that the operators should offer the same frequency in equilibrium, but this is not what happens. Instead, one operator tends to have a larger number of departures than the other. The frequency equilibrium is thus asymmetric, i.e. the operators’ frequencies are different from one another in equilibrium.

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1 For some parameter-values (though not in the base case), there are multiple Nash equilibria in addition to the symmetric points. In such cases, the equilibrium point with fewest departures is analysed.
The asymmetric frequency equilibrium is a consequence of operators having an incentive to lower their exposure to price competition. When operators have the same number of departures, all departures will be adjacent to departures controlled by a competitor. In contrast, if an operator has more departures than the other, a quasi-monopoly situation arises for some of its departures. When three consecutive departures are controlled by the same operator, the one in the middle is shielded from price competition (see Figure 1). In effect, every additional departure that the dominating operator adds will have this feature, creating an incentive for a dominant operator to increase its frequency further.

An entrant operator, by contrast, lacks such incentives. An operator with few departures will face a situation where any additional departure will face full price competition, and hence work to lower the general price level on the market. The ‘marginal departure’ is hence less profitable for the smaller operator, giving incentives for it to keep the number of departures low.

Of course, it is only profitable for the dominant firm to add more departures up to a point. With more departures, the average profit per departure tends to fall, because of declining average demand per departure. Even when an extra departure is profitable on its own, it induces losses on existing departures by attracting demand from them. This induced loss of new departures affects the larger operator the most. At some point, the induced loss will be greater than the profit of the extra departure, even if that departure is shielded from price competition. For this reason, one operator does not necessarily end up running all departures.

The phenomenon described here appears even though, as mentioned above, the operating cost per departure is assumed to be constant. In reality, there are economies of scale in the number of services, which will benefit a larger operator (Wheat & Smith, 2015) which enhances the frequency asymmetry further. Similarly, a long-standing good reputation among customers by the incumbent operator might lead to it charging higher fares and thus becoming more profitable than its competitor, as suggested by Fröidh & Byström (2013) and Ruiz-Rúu & Palacín (2013), also enhancing the frequency asymmetry. The appearance of asymmetry of frequencies despite symmetric preconditions is analogous to the results of a simulation of the airline market made by Schipper, Nijkamp & Rietveld (2007).

The two competing operators run 25 trains per day between them, compared to 22 in the welfare-maximising benchmark (see Table 3), while average fares are roughly three times higher and consumer surplus 30% lower. Aggregate welfare is 13% lower in duopoly compared to welfare-maximum. This compares favourably to the welfare difference of 30% between profit-maximising monopoly and welfare maximum. This welfare gain, equivalent to 17% of the theoretical unsubsidised welfare-maximum, constitutes the main argument for...
introducing open access competition on passenger railway lines. Compared to the profit-maximising monopoly, the welfare gain is 24% when open access competition is introduced, if it results in duopoly Nash equilibrium.

Table 2. Net profits (in SEK per day) of incumbent and entrant for different combinations of departure frequencies. The point 16:0 is the profit-maximising frequency without the entrant; the point 18:7 is the Nash equilibrium in duopoly.

<table>
<thead>
<tr>
<th>Incumbents’ number of departures</th>
<th>Entrant no. of dep.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incumbent</td>
<td>14</td>
<td>3,790</td>
<td>3,800</td>
<td>3,804</td>
<td>3,803</td>
<td>3,797</td>
<td>3,788</td>
<td>3,777</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Incumbent</td>
<td>1</td>
<td>3,327</td>
<td>3,343</td>
<td>3,352</td>
<td>3,357</td>
<td>3,367</td>
<td>3,353</td>
<td>3,346</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>1</td>
<td>-216</td>
<td>-233</td>
<td>-249</td>
<td>-264</td>
<td>-276</td>
<td>-289</td>
<td>-300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incumbent</td>
<td>2</td>
<td>2,882</td>
<td>2,902</td>
<td>2,914</td>
<td>2,925</td>
<td>2,930</td>
<td>2,930</td>
<td>2,922</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>2</td>
<td>33</td>
<td>1</td>
<td>-26</td>
<td>-54</td>
<td>-78</td>
<td>-100</td>
<td>-119</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incumbent</td>
<td>3</td>
<td>2,456</td>
<td>2,411</td>
<td>2,430</td>
<td>2,491</td>
<td>2,500</td>
<td>2,503</td>
<td>2,503</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>3</td>
<td>241</td>
<td>259</td>
<td>237</td>
<td>131</td>
<td>96</td>
<td>67</td>
<td>41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incumbent</td>
<td>4</td>
<td>2,057</td>
<td>2,053</td>
<td>2,068</td>
<td>2,083</td>
<td>2,122</td>
<td>2,130</td>
<td>2,108</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>4</td>
<td>395</td>
<td>350</td>
<td>313</td>
<td>271</td>
<td>235</td>
<td>198</td>
<td>194</td>
<td></td>
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<tr>
<td>Incumbent</td>
<td>5</td>
<td>1,717</td>
<td>1,697</td>
<td>1,701</td>
<td>1,716</td>
<td>1,762</td>
<td>1,766</td>
<td>1,766</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>5</td>
<td>462</td>
<td>420</td>
<td>391</td>
<td>351</td>
<td>321</td>
<td>289</td>
<td>262</td>
<td></td>
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<tr>
<td>Incumbent</td>
<td>6</td>
<td>1,399</td>
<td>1,378</td>
<td>1,405</td>
<td>1,410</td>
<td>1,449</td>
<td>1,442</td>
<td>1,404</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>6</td>
<td>504</td>
<td>449</td>
<td>396</td>
<td>367</td>
<td>353</td>
<td>330</td>
<td>287</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incumbent</td>
<td>7</td>
<td>1,118</td>
<td>1,101</td>
<td>1,158</td>
<td>1,167</td>
<td>1,170</td>
<td>1,169</td>
<td>1,162</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>7</td>
<td>509</td>
<td>443</td>
<td>421</td>
<td>385</td>
<td>355</td>
<td>328</td>
<td>304</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incumbent</td>
<td>8</td>
<td>924</td>
<td>919</td>
<td>908</td>
<td>922</td>
<td>930</td>
<td>932</td>
<td>931</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant</td>
<td>8</td>
<td>493</td>
<td>430</td>
<td>404</td>
<td>366</td>
<td>332</td>
<td>302</td>
<td>276</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

So far in our analyses, we have made the conventional assumption that operator profits are somehow returned to the society at large, for example through corporate taxes and shareholder dividends. We will now look at where the profits are going. Under a regulated monopoly, the sole operator is often a government-owned company and its profits hence accrue to the state and ultimately to taxpayers. Under a competitive regime, however, operators may be private and foreign owned. From the point of view of the citizens of a country, profits may hence be “lost” when they accrue to foreign companies rather than the government or domestically owned companies. It is therefore interesting to see how total welfare (consumer surplus and profits) under monopoly compares with consumer surplus (i.e. excluding profits) in duopoly. In our simulation, consumer surplus in the duopoly Nash equilibrium is almost exactly the same (0.7% lower) as the total welfare in profit-maximising monopoly. This implies that if profits are transferred abroad following deregulation, the domestic welfare gain vanishes.

Table 3. Duopoly under Nash equilibrium, compared to two monopoly situations and a benchmark with low barriers to entry (“Multiple operators”).

<table>
<thead>
<tr>
<th>Welfare max., no subsidy</th>
<th>Profit max.</th>
<th>Duopoly, Nash eq.</th>
<th>Multiple operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of departures</td>
<td>22</td>
<td>16</td>
<td>18 + 7</td>
</tr>
<tr>
<td>Average fare</td>
<td>104</td>
<td>703</td>
<td>11,100</td>
</tr>
<tr>
<td>No. of passengers</td>
<td>13,300</td>
<td>7,100</td>
<td>11,100</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>8,840,000</td>
<td>2,430,000</td>
<td>6,190,000</td>
</tr>
<tr>
<td>Producer surplus</td>
<td>0</td>
<td>3,800,000</td>
<td>1,520,000</td>
</tr>
<tr>
<td>Total welfare</td>
<td>8,840,000</td>
<td>6,230,000</td>
<td>7,720,000</td>
</tr>
</tbody>
</table>

Profits make up a large share of total welfare under profit-maximising monopoly. If revenues are used to replace distortionary taxes, profits should arguably be marked up by the marginal cost of public funds (MCPF) in order to reflect their overall welfare effect. Calculated this way, with MCPF=1.3, consumer surplus in duopoly is 16% lower than total welfare under monopoly.

If capacity is expensive and inflexible, it is possible that one operator will choose a different frequency from what is implied by the Nash equilibrium in order to force its competitor into a position more favourable to itself, under the logic of a Stackelberg game. For some parameter values, though not the base case, there is a
Stackelberg equilibrium\(^6\) that is different from the Nash equilibrium. Noteworthy is that the Stackelberg equilibrium has fewer (or equally many) departures compared to the Nash equilibrium, and both profits and fares are higher (or equal). This is not typical; Stackelberg games often result in more competitive outcomes than do Nash games (see e.g. Maskin and Tirole (1987)). The special characteristics of the Stackelberg equilibrium in this paper is an effect of the asymmetry of frequencies described above: When the incumbent offers fewer departures, the two operators move toward symmetry in frequency space, thus (temporarily) increasing competition and reducing profits. The entrant’s best response is to move away from the diagonal, i.e. to decrease its own frequency as well. Total welfare is usually lower in the Stackelberg equilibrium than in the Nash equilibrium, unless the two are equal.

4.3. Ordering and timing of departures

As mentioned above, we assume that operators only decide frequencies, while the infrastructure manager decides precise departure times, i.e. the ordering and timing of departures, and we assume that the infrastructure manager strives to maximise aggregate social welfare. This is obviously a simplification of reality, but it reflects the fact that under open access competition, operators cannot freely choose departure times: they apply for capacity and the infrastructure manager (which is normally a government agency) determines how to construct a feasible timetable out of operators’ (possibly competing) requests. The timing and ordering of departures has substantial impact on demand, fares and overall welfare. Assuming that the objective of the regulator is to maximise social welfare, we will demonstrate that the regulator should spread departures evenly (assuming a uniform PDT distribution), and aim for maximal competition between operators by mixing departures by different operators. That departures are spread evenly is of course a consequence of our simplified assumption of a uniform PDT distribution, but the finding that departures run by competing operators should be mixed as much as possible is an insight with real-world implications. The intuition is that this minimises the local quasi-monopoly power that arises whenever the same operator controls adjacent departures.

Table 4. Welfare effect of clustering rather than intermingling departures.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total welfare</td>
<td>5,540,000</td>
<td>5,200,000</td>
<td>-6.1%</td>
</tr>
<tr>
<td>Combined profit</td>
<td>2,550,000</td>
<td>2,670,000</td>
<td>+4.8%</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>2,990,000</td>
<td>2,530,000</td>
<td>-15.4%</td>
</tr>
<tr>
<td>Total demand</td>
<td>7,600</td>
<td>7,000</td>
<td>-8.3%</td>
</tr>
<tr>
<td>Average fare</td>
<td>497</td>
<td>560</td>
<td>+12.7%</td>
</tr>
</tbody>
</table>

Table 4 illustrates how the ordering of departures affects results by comparing two versions of point 4:2 in frequency space (i.e. with four incumbent and two entrant departures): one that is maximally intermingled and one with clusters of departures run by the same operator. The timetable with intermingled departures yields higher total welfare. As expected, this is driven by an intensified price competition that decreases fares, increases demand and improves consumer surplus while lowering profits, compared to the clustered scenario.

Table 5. Welfare effect of unequally spread departures.

<table>
<thead>
<tr>
<th></th>
<th>Consumer surplus</th>
<th>Total welfare</th>
<th>Welfare change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline: even spacing</td>
<td>2,990,000</td>
<td>5,540,000</td>
<td>-</td>
</tr>
<tr>
<td>Move dep. 1</td>
<td>2,990,000</td>
<td>5,570,000</td>
<td>+0.5%</td>
</tr>
<tr>
<td>Move dep. 2</td>
<td>2,960,000</td>
<td>5,380,000</td>
<td>-2.9%</td>
</tr>
<tr>
<td>Move dep. 3</td>
<td>2,990,000</td>
<td>5,400,000</td>
<td>-2.6%</td>
</tr>
<tr>
<td>Move dep. 4</td>
<td>2,920,000</td>
<td>5,360,000</td>
<td>-3.2%</td>
</tr>
<tr>
<td>Move dep. 5</td>
<td>2,960,000</td>
<td>5,380,000</td>
<td>-2.9%</td>
</tr>
<tr>
<td>Move dep. 6</td>
<td>2,940,000</td>
<td>5,270,000</td>
<td>-4.9%</td>
</tr>
</tbody>
</table>

The case for evenly distributed departures is illustrated in Table 5, which shows the welfare effects of changing departure times slightly. It also uses point 4:2 in frequency space. One departure at a time is moved forward in time 50% of the distance to the nearest departure. The table shows the effect on aggregate welfare of

\(^6\) In a Stackelberg-game the independent variables are normally quantity and price; here they are frequency and price.
each change, compared to the base case where departures are spread evenly. The effect is small but clear: any deviation from the situation where departures are spread evenly reduces total welfare (except on the boundary).

4.4. Multiple operators

Up to now, we have only studied two potential operators. Most real open access markets for passenger traffic do in fact only have two or at most three competing operators, for example Italy (Beria, Redondi, & Malighetti, 2014), the Czech Republic (Zdenek, Kvizda, Jandová, & Rederer, 2016) (Zdenek, Kvizda, Nigrin, & Seidenglanz, 2014) and Sweden (Vigren, 2016). Preston (2008) cites too thin demand in most markets along with economies of scale and density as reasons to assume that the number of actors in this type of market will be very limited. This is also in line with the driving force of the asymmetry of frequencies as discussed in section 4.2.

We will now relax the requirement on number of operators and instead assume that nothing in particular restricts it, such as large barriers to entry. Instead, both entrants and incumbents will add or remove departures depending on how their total profitability is affected on the margin.

As discussed in section 4.2, new departures decrease the profitability of existing departures, as there will be fewer passengers per departure (or lower fares to compensate). As this affects incumbents but not new entrants, the marginal profit of adding an extra departure (the profit of that departure minus lost profit of existing departures) is lower for an incumbent than for an entrant. Assume that no operator is large enough to form pseudo-monopoly situations as described in section 4.2, and that instead the smallest operator always has the least to lose from adding new departures. In equilibrium, each operator will then run a single departure, and the number of departures (and operators) becomes the highest possible that permit them all to be profitable.

Using the parameters of our base case, there will be six operators, lowering combined frequency by 73% compared to the welfare maximising benchmark (76% compared to the Nash equilibrium). Total welfare decreases by 63% (57%) by the same comparison, and profits fall by over 62% compared to the Nash equilibrium (see Table 3). Duopoly under Nash equilibrium, compared to two monopoly situations and a benchmark with low barriers to entry ("Multiple operators"). Note however that numbers in this scenario vary greatly with the value of operators’ fixed cost κ.

5. Implications for policy market regulation

In summary, our simulation results show that going from a monopoly to open access competition tends to increase social welfare substantially, reducing profits and benefitting travellers. Indeed, total welfare in the duopoly situation is close to the ideal, hypothetical case with a welfare-maximising operator under a cost recovery constraint.

A welfare-maximising infrastructure agency should arrange operators’ departures so that they are intermingled, in order to increase price competition. However, operators in duopoly will not offer equally many departures in equilibrium. This asymmetric frequency equilibrium lowers total welfare compared to symmetry, so it is worth asking whether there are effective market regulations to force operators to offer similar frequencies.

The analyses assume that all producers’ surplus are included in the social welfare. However, this assumption is not innocuous. In practice, monopolies often consist of a government-owned company, meaning that profits accrue to tax payers, while under duopoly, at least some operators may be foreign owned. Hence, a national policy maker may well consider profits accruing to foreign shareholders as “lost” from a domestic perspective. It should be stressed that railway markets are different from standard product markets since there is only physical room for a limited number of producers, and each departure time presents a certain monopoly power. This means that even in the long run and under the assumption that the railway market is otherwise perfect, firms will make a profit, which can be substantial. This is in contrast to standard product markets where long-run profits can reasonably be assumed to be zero when capital costs etc. are taken into account. This means that the question of to whom profits accrue is not moot. If operators’ profits are regarded as “lost” in the duopoly situation, this tends to cancel out most of the increase in social welfare in the simulations (although the precise loss depends on parameter values). Hence, it is worth asking whether there are policies or regulations that recoup at least some of operators’ profits to the government (and hence the taxpayer collective).

Another significant problem is the fact that the combined profits of the operators are always lower than the profit of a monopolist. This means that operators have an incentive to restore the monopoly situation, for example by merging or colluding, or by one operator buying the other’s departure slots (if this is allowed). The increased profit will be more than enough to compensate the competing operator. Hence, if departure slots can be traded, or if operators can merge or buy each other, or collude in other ways (for example by agreeing to divide

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7 Without this assumption, a possible outcome is the entry deterrence Nash equilibrium as described in section 4.1.
the overall railway market between them), the competitive situation is not stable, but will revert to monopoly. The fundamental reason for this is that for any fixed time period, there is a finite number of departure slots, so the number of possible entrants is limited in any given time-period. There is hence a commercial incentive for one operator to buy or somehow gain control of all the slots to become a monopolist. Once an operator has control over all departure slots, the monopolist may choose not to use all of them.

Constructing a procedural framework that prevents this outcome is not trivial. One countermeasure is to forbid operators to trade departure slots with each other. But such a ban may not be enough: negotiations and some amount of quid-pro-quo between operators is usually a necessary part of timetable construction in reality. Moreover, tacit collusion between operators to divide submarkets between them may be very difficult to prevent in practice.

This observation also poses a significant problem for procedures aiming to allocate scarce capacity through market mechanisms, such as auctions or scarcity pricing. Since the profits of a monopolist is higher than the combined profits of two competing operators, a would-be monopolist will always be willing to outbid its competitors. Hence, it is highly likely that a capacity allocation (with a finite number of departure slots) based on operators’ willingness-to-pay will result in a monopoly situation – with the welfare losses highlighted above.

We have used our model to study a number of policy proposals to counteract the problems above. However, several of them have in fact turned out to be either ineffective or counter-productive, and have therefore not been included in the above proposal. These include measures to curb excessive operator profits and to force the frequency equilibrium toward symmetry.

First, consider the case to curb excessive profit. In a duopoly market, competition is less than perfect and operators’ profits do not decrease towards zero, as discussed above. Policy makers may therefore wish to tax those earnings. One option to their disposal is to raise infrastructure charges. We have looked at the effects of a flat fee per departure that is high enough to lower total profits by around a third.

The problem with raising infrastructure charges to curb profit is that, to succeed, it must amount to a substantial increase in the cost per departure, thus altering the incentives of the frequency game. Therefore, the equilibrium switches to a point with fewer departures and higher fares, resulting in a welfare loss that is well above what the government earns from the charge increase. This inefficiency of infrastructure charges above marginal costs and externalities is unsurprising. In fact, it holds generally that it is optimal to operate an economy at the production-possibilities frontier, implying that intermediary goods – including infrastructure – should not be taxed. Diamond and Mirrlees (1971) show that this result generally holds even in the presence of distortionary taxation.

Now consider the frequency equilibrium. While the above policy recommendations are shown to result in asymmetric frequency equilibrium, the welfare maximising frequency combination under price competition is symmetric. It would be better for society if operators could be made to offer an equal number of departures, unless the means to make such outcome come about is so inefficient that it cancels out the advantage. This turns out to be the case: a number of policies designed to produce equally many departures end up either failing through not providing sufficient incentives for the smaller operator to raise frequency, or damaging total welfare by lowering combined frequency or raising average fares.

One possibility is for the infrastructure agency to decline requests for capacity beyond a certain point in order to reduce the frequency of the incumbent. However, this is in effect similar to forcing the incumbent to pursue a Stackelberg strategy (described in section 4.2). According to the logic of asymmetric frequency equilibria, the entrant then has incentives to also decrease frequency, contrary to the regulator’s intentions. Fares increase, ridership drops and consumer surplus decreases because of the policy. Profits may increase under such a scheme, but not enough to keep social welfare from falling.

Another possibility is to have rules that force operators to have a certain minimum frequency. The dynamics of the price game is unaffected by this. However, the regulating agency may not know what frequency is optimal, as it lacks information needed to calculate the value of individual departures, such as ridership and fares. In addition, forcing operators to run more departures than they had planned for may result in a service of poor quality, or not be feasible at all because of the long lead-times and large financial obligations associated with acquiring rolling stock and scaling up operations.

Yet another option is to skew incentives to make the offering of equally many departures seem more attractive to operators. The regulator might even abstain from trying to maximise competition, in cases where the competitors offer close to equal frequencies. Instead, it bundles the entrant’s departures closer together, thus easing the price pressure there. The priority should be to lessen the burden on the entrant while preserving the pressure on the incumbent as much as possible, in order to force a new Nash equilibrium with lower combined profits compared to the previous equilibrium point. The problem with this idea is that while the incumbent seems to gain from a reordering of departures to lessen price competition, the entrant does not. The intuition for this is that without drastic changes, departures belonging to the entrant will continue to lie close to the competitor’s departures. For the incumbent, in contrast, its existing shielded pseudo-monopoly areas can easily be expanded.
Simulations confirm that any changes large enough to have a positive impact on the behaviour of the entrant will increase average fares enough to make the welfare effect of the policy negative.

Yet another idea is to vary access charges according to the number of departures that an operator runs. It seems however that this will not work without dramatic variations that are probably unfeasible due to practicality and fairness reasons.

6. Conclusions

Traditionally, most railway markets have been monopolies. There are different types of monopolies, for example public utilities, vertically integrated commercial firms, commercial firms that operate under concession and commercial firms that uphold a monopoly position in an unregulated environment. These various regimes correspond to different operator objective functions, including welfare maximisation (with some constraints, e.g. on cost); pure profit maximisation; and profit maximisation under an entry deterrence strategy.

Open access competition has the potential to increase overall welfare by reducing the deadweight losses stemming from monopoly situations. Our simulation model demonstrates that there exists a stable equilibrium with two operators, where both operators make positive profits. The duopoly situation benefits travellers at the expense of operators, compared to profit-maximising monopoly. Total welfare increases substantially (around 24% in our simulations), and is in fact close to unsubsidised welfare maximum. One operator tends to offer higher frequency and higher average fares than the other, despite the two operators being modelled as identical. The entry deterrence scenario also leads to higher frequency compared to pure profit-maximising monopoly, which leads to a small positive welfare contribution.

The timing and ordering of departures affects social welfare. A welfare maximising infrastructure manager should use its regulatory power to ensure a high level of price competition by placing competing operators’ departures adjacent to each other, rather than letting a departure enjoy a local monopoly in time by surrounding it with departures by the same operator. This gives travellers options to choose from throughout the day, thus increasing competition. Departures should also be spread evenly, not lumped together, in order to maximise welfare. (Frequencies should not necessarily be constant throughout the day however, as that result rests on an assumption of uniform PDT distribution.)

There are incentives for one operator to buy the other’s departure slots in order to create a monopoly situation. The source of this is ultimately that there is a finite number of profitable departure slots, because of fixed costs per departure. This result appears even without economies of scale in production. In reality, such economies exist and will tend to reinforce this problem.

In all, this paper provides insights concerning the dynamics of railway markets with open access competition, including an estimate of the welfare gains from replacing profit-maximising monopoly with competition, as well as a rough estimate of potential gains from introducing such market regime, and advice to regulators to set timetables that increases competition, and take measures to prevent behaviour by market participants that diminishes competition.

7. Acknowledgement

Funding from the Swedish Transport Administration is gratefully acknowledged. The authors have benefited from fruitful discussions with Abderrahman Ait-Ali, Martin Aronsson, Chris Nash, Andrew Smith and Victoria Svedberg.
Appendix A. Calculating consumer surplus

Use the equation

\[ D(t) = \varphi(t) - \beta c(t) \]

Solve for the generalised cost:

\[ c(t) = \frac{\varphi(t) - D(t)}{\beta} \]

For a given PDT \( t \), the consumer surplus is the triangular area

\[ CS(t) = \frac{1}{2} D(t) \left( \frac{\varphi(t) - D(t)}{\beta} \right) = \frac{D(t)^2}{2\beta} = \]

\[ = \frac{1}{2\beta} (\varphi - \beta c(t))^2 = \frac{\varphi^2}{2\beta} + \frac{\beta}{2} c(t)^2 - \varphi c(t) = \]

\[ = \frac{\varphi^2}{2\beta} + \frac{\beta}{2} (p_n + \alpha |T_n - t|)^2 - \varphi(p_n + \alpha |T_n - t|) = \]

\[ = \frac{\varphi^2}{2\beta} + \frac{\beta}{2} p_n^2 + \frac{\alpha^2 \beta}{2} (T_n - t)^2 + \alpha \beta p_n |T_n - t| - \varphi p_n - \varphi \alpha |T_n - t| = \]

\[ = \frac{\varphi^2}{2\beta} + \frac{\beta}{2} p_n^2 - 2 \varphi \beta p_n + \frac{\alpha^2 \beta}{2} (T_n - t)^2 + \alpha (\beta p_n - \varphi) |T_n - t| = \]

\[ = \frac{1}{2\beta} (\varphi - \beta p_n)^2 + \frac{\alpha^2 \beta}{2} (T_n - t)^2 + \alpha (\beta p_n - \varphi) |T_n - t| \]

Integrate over \([\tau_n, \tau_{n+1}]\) assuming \( \varphi(t) \) is constant (as in the simulation):

\[ CS_n = \frac{(\varphi - \beta p_n)^2}{2\beta} \int_{\tau_n}^{\tau_{n+1}} dt + \frac{\alpha^2 \beta}{2} \int_{\tau_n}^{\tau_{n+1}} (T_n - t)^2 dt + \alpha \beta p_n - \varphi \int_{\tau_n}^{\tau_{n+1}} (t - T_n) dt \]

\[ + \alpha (\beta p_n - \varphi) \int_{\tau_n}^{\tau_{n+1}} (t - T_n) dt \]

\[ = \frac{(\varphi - \beta p_n)^2}{2\beta} (\tau_{n+1} - \tau_n) + \frac{\alpha^2 \beta}{2} ((T_n - \tau_n)^3 + (\tau_{n+1} - T_n)^3) \]

\[ + \alpha \beta p_n - \varphi \left( (T_n - \tau_n)^2 + (\tau_{n+1} - T_n)^2 \right) \]
Appendix B. Calculating demand

As before,

$$\tau_n = \frac{p_{n+1} - p_n}{2\alpha} + \frac{T_{n+1} + T_n}{2}, n = 1, \ldots, N - 1$$

Assuming that $\varphi(t)$ is constant, total demand becomes

$$D = \sum_n \left\{ \int_0^{\tau_n} (\varphi - \beta(a\Delta + p_n))d\Delta + \int_{\tau_n}^{\tau_{n+1}} (\varphi - \beta(a\Delta + p_n))d\Delta \right\}$$

$$= \sum_n \left\{ (\varphi - \beta p_n)(\tau_{n+1} - \tau_n) - \frac{\alpha\beta}{2} ((T_n - \tau_n)^2 + (\tau_{n+1} - T_n)^2) \right\}$$
Appendix C. Sensitivity analysis

Robustness tests have been conducted by varying the parameters α, β and κ. Each variable has been multiplied by a coefficient of 0.5, and 1.5 relative to the base case. The table below presents the key results. As seen, predicted relative welfare differences between the welfare-maximising benchmark, profit-maximising monopoly and Nash equilibrium duopoly are quite stable over a range of parameter values. Changes in demand are also stable, while the change in service frequency (number of departures) varies more and fares vary much more, especially with demand/price elasticity.

<table>
<thead>
<tr>
<th></th>
<th>Base case</th>
<th>α +50%</th>
<th>α -50%</th>
<th>β +50%</th>
<th>β -50%</th>
<th>κ +50%</th>
<th>κ -50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>6,000</td>
<td>9,000</td>
<td>3,000</td>
<td>6,000</td>
<td>6,000</td>
<td>6,000</td>
<td>6,000</td>
</tr>
<tr>
<td>β</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
<td>-15</td>
<td>-5</td>
<td>-10</td>
<td>-10</td>
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No.
of
departures
profit max./welfare max.
Nash eq./welfare max.
Average fare
profit max./welfare max.
Nash eq./welfare max.
Demand
profit max./welfare max.
Nash eq./welfare max.
Social welfare
profit max./welfare max.
Nash eq./welfare max.
References


